Problem solving guidance

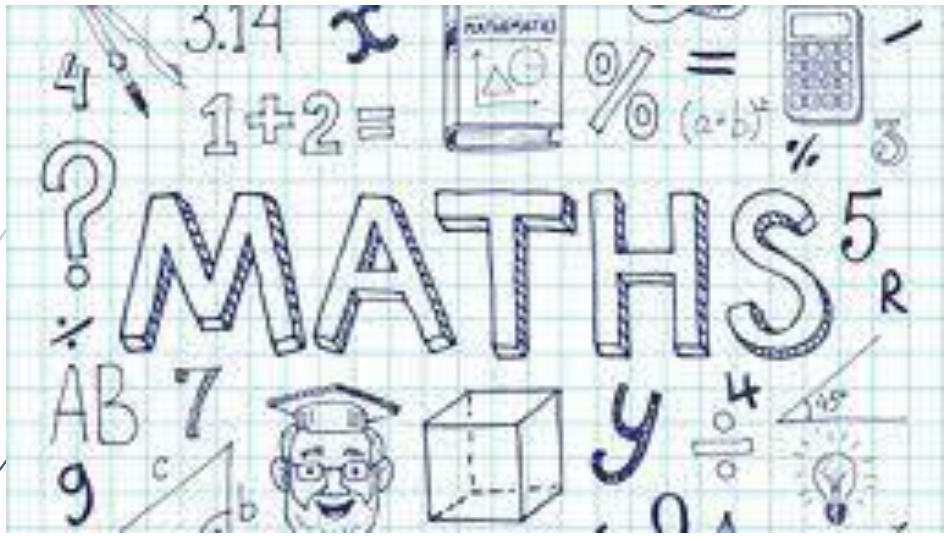
Finding all possibilities

EYFS KSI

Lower KS2

Upper KS2

Autumn 1



Mathematics Intent

At Teagues Bridge, our intention is **ambitious.** We aim to create strong mathematicians who have the necessary skills and understanding to tackle mathematical challenges in varying contexts, including the ability to reason and apply their knowledge to solving problems. This should mean that children are able to apply their knowledge to everyday life and can **aspire** to achieve anything that they want. We want our pupils to have strong mental manipulation and to use written strategies when appropriate.

Our philosophy for mathematics is replacing an idea that maths is lots of rules and numbers with a study of patterns and connected ideas. In early years they will build a foundation of number understanding and representation through mainly concrete and pictorial representations. The approach will be supported by in depth questioning, throughout the school to develop mastery.

Use of CPA is encouraged to ensure the curriculum is accessible for all children and that they all have the **opportunity** and are able to demonstrate their understanding in a variety of ways. This will enable them to have a good understanding of maths and not just the ability to follow a procedure. We want to **empower** them to want to ask questions and want to find the answers.

Aims: The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and nonroutine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

Our lessons are structured to enable all children to achieve and have an **opportunity** to make progress with their learning. Each lesson begins with a **CLIC maths** activity, where they have chance to develop their mental strategies, secure number facts and number manipulation. They then **develop** their mathematical fluency with the teacher modelling and explaining before they have a go themselves. Children then have a **reasoning/ problem solving** activity which is a variation of the previous work to demonstrate they have mastered the objective. Children who are ready can then **challenge** themselves with a task that requires applying the learning to a greater depth. We have our own programme of study which is supported with schemes like White Rose to support.

EYFS and Key Stage one – Finding all possibilities

Finding All Possibilities	EYFS (conditional)	KS1 (conditional)
I know how and when to work systematically. I know how and when to check for repeats. I know how to satisfy the criteria with solution/s.	I know when to put items and objects, including pictures in order. I know when items are the same.	I know when I will put my answers in order. I know what resources to use. I know if I have some answers the same.

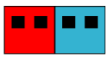
Key Skills and Strategy Development EYFS		Question stems
Work systematically.		How will I put my objects in order? What resources shall we choose?
Follow a set of instructions.		What am I allowed to do?
I know when items/objects are the same.		Have I solved the problem?
Example problems		**Declarative, procedural knowledge**

In a certain city houses had to be built in a particular way.

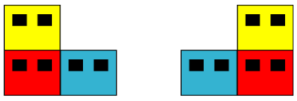
There had to be two rooms on the ground floor and all other rooms had to be built on top of these.

Families were allowed to build just one room for each person living in the house.

So a house for two people would look like this:

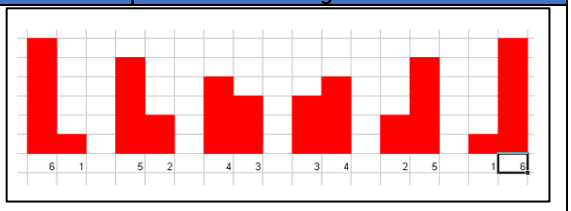


but a house for three people could look like one of these:



There are some families of seven people living in the town.

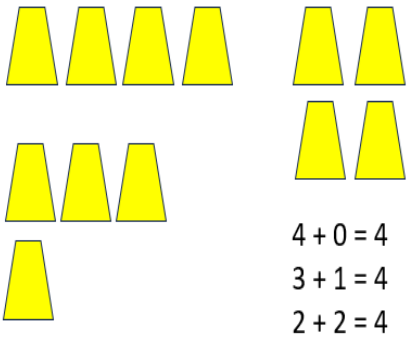
In how many different ways can they build their houses?



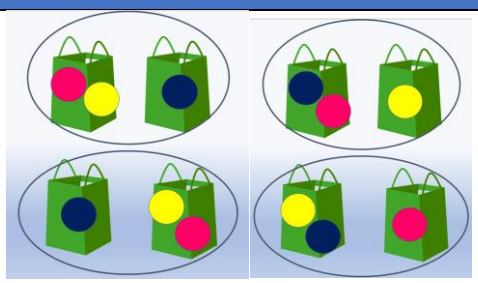
Declarative: Early years: numbers and number bonds to 10; concepts and vocabulary for talking about maths and mathematical patterns (size, weight, capacity, quantity, position, distance, time)

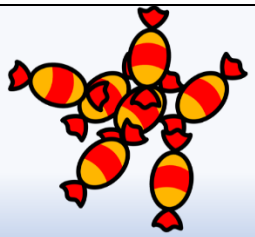
Procedural: Early years: accurate counting, single digit addition and subtraction, halving doubling and sharing

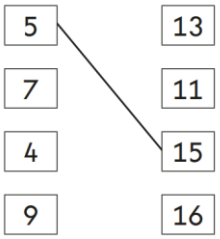
How many different ways can we build the 4 sandcastles?

Keystage 1 – Finding all possibilities

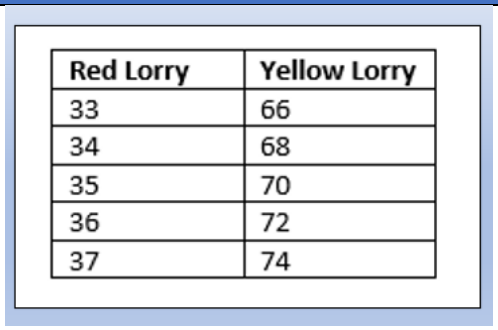
Key Skills and Strategy Development KS1	Question stems
Work systematically.	How will I put my answers in order? What resources shall we use?
Check for repeats.	Are some of my answers the same?
Satisfy the criteria with solution/s.	Have I answered the question? Solved the problem?
Declarative knowledge	**Procedural knowledge**
Concepts, representations, and associated vocabulary: ⇒ simple fractions ⇒ basic arithmetic: the numbering system and its symbols, place value, conventions for expressions and equations, counting, addition, subtraction, equal sharing, doubling, balancing simple arithmetic equations, classifying numbers (odd, even, teens), inverse operations, estimation, numerical patterns.	Efficient and accurate methods: ⇒ counting up and down in 1s, 2, 5s, 10s and 1/2s; addition; subtraction, equal sharing, division and multiplication ⇒ reading, writing of the digits/symbols, vocabulary and phrases required for working with simple fractions, arithmetic expressions and equations, numerical patterns ⇒ construction and interpretation of categorical data:, charts, tables
Example problems	
This challenge involves three beads and lots of bags. There are as many bags as you need. Find a way to put the beads into some of the bags. Find another, different way to put the beads into some of the bags. How many different ways can you make?	 Y1 – Number bonds to 3, represent and use number bonds and related subtraction facts within 2

There are 15 sweets altogether and 3 bags. There are 7 sweets in one bag. How many sweets could be in the other 2 bags? Can you find some of the different ways they could be shared between the bags? 	7 ? ? 1 7 2 6 3 5 4 4 Y2 – number bonds to 20 and within, recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
Whitney has this many cubes in one hand.  She has fewer cubes in the other hand. How many cubes could she have in her other hand?	Y1 – less than 5, represent and use number bonds and related subtraction facts within 20
There are 2 ice-creams and up to 16 marshmallows. How many ways can you share marshmallows between the 2 ice-creams so that the 2nd ice-cream has double the amount of marshmallows as the 1st ice-cream? 	Y2 – doubles and halves to 20, recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
	Y1 – number bonds to 10 and within, represent and use number bonds and related subtraction facts within 2

Stanley found the following number bonds to 8 3 + 5 0 + 8 1 + 7 4 + 4 2 + 6 What order would Stanley have found them in if he'd have worked systematically?	
14 Match **all** the pairs of numbers that add to **20** One is done for you.  1 mark	Y1/2 – number bonds to 20, represent and use number bonds and related subtraction facts within 2 recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
31 Sita solved this calculation. 16 – 4 = 12 Circle **all** of the calculations that show how Sita could check her answer. 16 + 4 4 + 12 4 + 16 12 + 4 1 mark	Y1/2 Number bonds to 2 and within, represent and use number bonds and related subtraction facts within 2 Recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100.

Lower Key Stage 2 – Finding all possibilities

Finding All Possibilities	EYFS (conditional)	KS1 (conditional)	LKS2 (conditional)
I know how to work systematically. I know how to check for repeats. I know how to satisfy the criteria with solution/s.	I know when to put items and objects, including pictures in order. I know when items are the same.	I know when I will put my answers in order. I know what resources to use. I know if I have some answers the same.	I know the best way to record the results. I know if some solutions repeated. I know if I have solved the problem and when there is more than one solution.

Key Skills and Strategy Development LKS2	Question stems
Work systematically. Where is the starting point?	What is the best way to show the results? A table? A list?
Check for repeats.	Are some solutions repeated? Does this affect the outcome?
Satisfy the criteria with solution/s.	Have I answered the question? Solved the problem? Is there more than one solution?
Declarative knowledge	**Procedural knowledge**
Concepts, representations and associated vocabulary: ⇒ Arithmetic: enhanced knowledge of the code for number (to 1000s) including patterns and associated rules for addition and subtraction of numbers, decimal numbers, place value, negative numbers, associative and distributive laws ⇒ Maths facts: all multiplication facts for the 3, 4, 6, 7, 8, 9, 11, 12 multiplication tables, decimal equivalents of key fraction	Efficient and accurate methods: ⇒ counting up and down in multiples of 3, 4, 6, 7, 8, 9, 11, 12, 25, 50, 100, 1000, in tenths, in ones through to negative numbers ⇒ Column addition and subtraction ⇒ Mental addition and subtraction using patterns and rules of number ⇒ Short division and multiplication ⇒ Mental multiplication using derived facts
Example problems	Model answers
Red Lorry has travelled half the miles that Yellow Lorry has travelled. Red Lorry has travelled more than 32 miles and Yellow Lorry has travelled no more than 74 miles. Find **five** possible combinations of miles that the lorries could have travelled.	 Y3 – doubles and halves to 100

Red Lorry	Yellow Lorry
33	66
34	68
35	70
36	72
37	74

Chico's cards are all different.
Chico has chosen four cards that add up to 20. There is a number from 1 to 8 on each card.

What are they?

There are **seven different possibilities**. Try to find them all.

Card 1	Card 2	Card 3	Card 4
1	4	7	8
2	3	7	8
1	5	6	8
2	4	6	8
1	6	5	8
2	5	6	7
3	4	6	7

Y3 – number bonds to 20

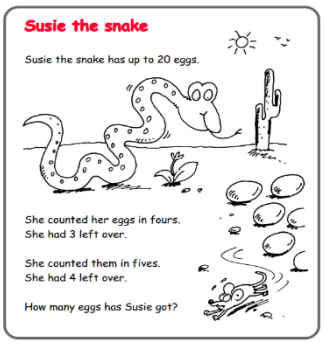
Using each digit card, which numbers can you make?

Use the place value grid to help.

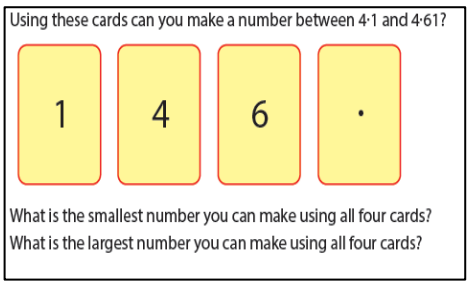
Hundreds	Tens	Ones

Compare your answers with a partner.

Y3 – 3 digit place value, recognise the place value of each digit in a three-digit number (hundreds, tens, ones).



Y4 – multiplication 4- and 5-times tables, recall multiplication and division facts for multiplication tables up to 12 × 12



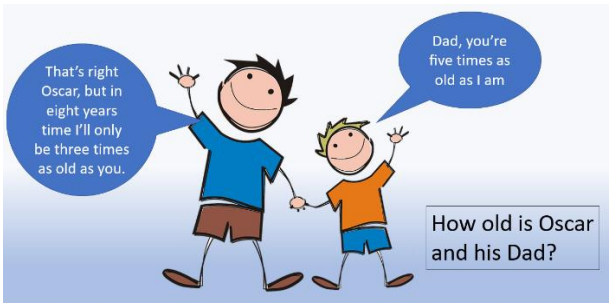
Y4 – decimal numbers, recognise and write decimal equivalents of any number of tenths or hundredths

Amy started with a 3 digit number. The number was > 200 but < 239, it was a multiple of 4 but not a multiple of 8. Find all the possible numbers.	204, 212, 220, 228, 236 Y4 – common multiples of 4 and 8, recall multiplication and division facts for multiplication tables up to 12 × 12, use place value, known and derived facts to multiply and divide mentally.

Upper Key Stage 2 – Finding all possibilities

Finding All Possibilities	EYFS (conditional)	KS1	LKS2	UKS2
I know how to work systematically. I know how to check for repeats. I know how to satisfy the criteria with solution/s.	I know when to put items and objects, including pictures in order. I know when items are the same.	I know when I will put my answers in order. I know what resources to use. I know if I have some answers the same.	I know the best way to record the results. I know if some solutions repeated. I know if I have solved the problem and when there is more than one solution.	I know when to identify are the starting and stopping points. I know when some solutions are repeated and when it affects the outcome. I know when the criteria restrict the number of possibilities.

Key Skills and Strategy Development UKS2	Question Stems
Work systematically. Identify the start and finish points to complete all possible solutions.	What is the best way to show the results? A table? A list? What are the starting and stopping points?
Check for repeats against the criteria in the problem.	Are some solutions repeated? Does this affect the outcome? What does the criteria allow for?
Satisfy the criteria with solutions or find an exact solution.	Have I answered the question? Solved the problem? Is there more than one solution? Is there an exact answer to be found?
Declarative knowledge	**Procedural knowledge**
Upper Key Stage 2 Concepts, representations, and associated vocabulary: ⇒ Enhanced knowledge of the code for number: up to and within 1 000 000, multiples, factors, decimals, prime number facts to 100, composite numbers, indexation for square and cubed numbers	Upper Key Stage 2 Efficient and accurate methods ⇒ Scaling, coordinate geometry in all four quadrants ⇒ Division with remainders as fractions, decimals and where rounding is needed
Example problems	Model answers
11 Ally chooses a whole number. When she multiplies her number by **4**, the answer is **less than 100** When she multiplies her number by **5**, the answer is **greater than 100** Write a number that Ally could have started with. 1 mark	Y5 – multiples of 4 and 5 to 100 identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers



Oscar	Oscar + 8	Dad	Dad + 8
4	12	20	28
5	13	25	33
6	14	30	38
7	15	35	43
8	16	40	48

Y5/6 – common multiples
identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers

15

$a + b = 14$

a and b are whole numbers.

a is a one digit number.

b is a two digit number.

Find four different possibilities for a and b.

a	b

Y5 - add and subtract numbers mentally.
Y6 - find pairs of numbers that satisfy an equation with two unknowns.

Jake's Number

It is less than 50.
It is a 2-digit number.
3 is a factor of this number.
The sum of its digits is one third of the number.
What is Jake's number?

Y5 - identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers

2 Adam chooses the colours for a new team shirt.

The shirt has **two** colours.

There are four colours to choose from: **yellow**, **blue**, **white** and **red**.

Write the **two** missing combinations.

The shirt could be:

- yellow and blue
- yellow and white
- yellow and red
- blue and white.

Y6 – enumerate possibilities of combinations of two variables.