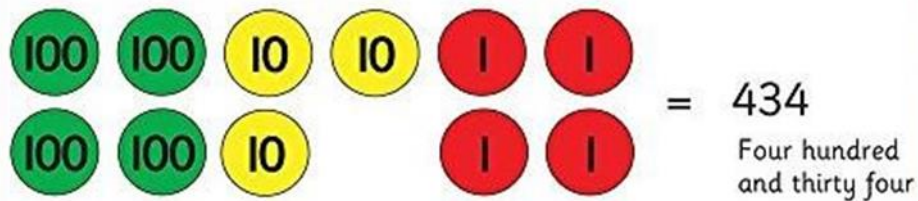
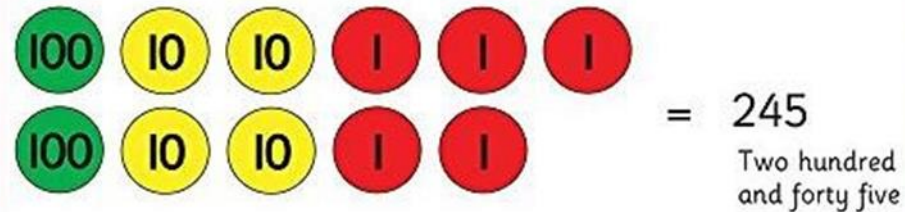


# Teagues Bridge Calculation Policy

Revised February 2023



Mathematics Intent

At Teagues Bridge, our intention is **ambitious**. We aim to create strong mathematicians who have the necessary skills and understanding to tackle mathematical challenges in varying contexts, including the ability to reason and apply their knowledge to solving problems. This should mean that children are able to apply their knowledge to everyday life and can **aspire** to achieve anything that they want. We want our pupils to have strong mental manipulation and to use written strategies when appropriate.

Our philosophy for mathematics is replacing an idea that maths is lots of rules and numbers with a study of patterns and connected ideas. In early years they will build a foundation of number understanding and representation through mainly concrete and pictorial representations. The approach will be supported by in depth questioning, throughout the school to develop mastery.

Use of CPA is encouraged to ensure the curriculum is accessible for all children and that they all have the **opportunity** and are able to demonstrate their understanding in a variety of ways. This will enable them to have a good understanding of maths and not just the ability to follow a procedure. We want to **empower** them to want to ask questions and want to find the answers.

**Aims:** The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and nonroutine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.


Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects.

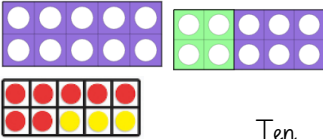
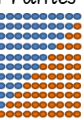


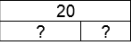
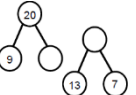

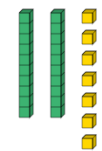
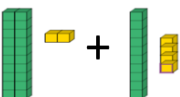
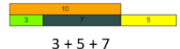
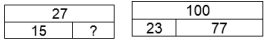
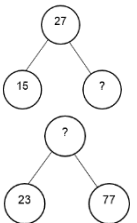
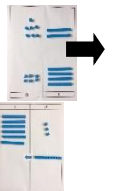
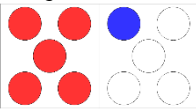
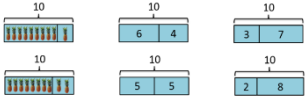

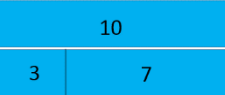
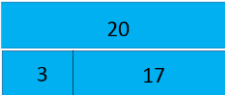

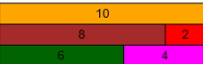
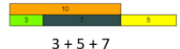
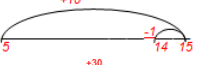


The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

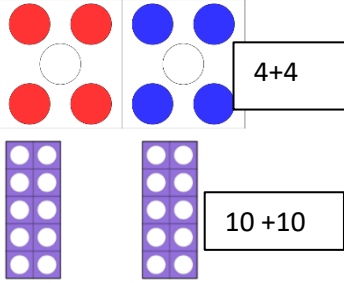
Our lessons are structured to enable all children to achieve and have an **opportunity** to make progress with their learning. Each lesson begins with a **CLIC maths** activity, where they have chance to develop their mental strategies, secure number facts and number manipulation. They then **develop** their mathematical fluency with the teacher modelling and explaining before they have a go themselves. Children


then have a **reasoning/ problem solving** activity which is a variation of the previous work to demonstrate they have mastered the objective. Children who are ready can then **challenge** themselves with a task that requires applying the learning to a greater depth. We have our own programme of study which is supported with schemes like White Rose to support.

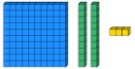


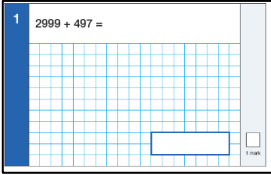
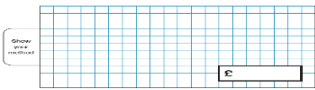
## Addition

EYFS	<b>Reception: ELG 2021</b>	
	<ul style="list-style-type: none"> <li>Have an understanding of number to 10, linking names of numbers, numerals, their value, and their position in the counting order.</li> <li>Subitise (recognise quantities without counting) up to 5.</li> <li>Automatically recall number bonds for numbers 0-5 and <i>for 10</i>, including corresponding partitioning facts.</li> <li>Automatically recall double facts up 5+5</li> <li>Compare sets of objects up to 10 in different contexts, considering size and difference.</li> <li>Explore patterns of numbers within numbers up to 10, including evens and odds.</li> </ul>	
<b>Year</b>	<b>1</b>	<b>2</b>
Layers of vocabulary  <b>Appendix Ia</b> Beck's Tiers of Vocabulary <b>Appendix Ib:</b> Vocabulary book	<p><b>Basic to subject specific (Beck's Tiers):</b> +, add, more plus make, sum, total altogether score double, near double one more, two more... ten more how many more to make...? how many more is... than...? how much more is...?</p> <p><b>Instructional vocabulary:</b> start from, start with, start at look at point, to show me</p>	<p><b>Basic to subject specific (Beck's Tiers):</b> +, add, addition, more, plus make, sum, total altogether score double, near double one more, two more... ten more... one hundred more how many more to make...? how many more is... than...? how much more is...?</p> <p><b>Instructional vocabulary:</b> tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of... show how you...</p>
NC 2014	Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.	Using concrete objects and pictorial representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods


	Concrete, pictorial, abstract		Concrete, pictorial, abstract			
Developing Conceptual/ Procedural Understanding	<p><b>Number bonds</b></p>  <p>Ten</p> <p><b>Frames</b></p>  <p> <math>2 + \square = 10</math>    <math>10 - \square = 3</math>  <math>5 + \square = 10</math>    <math>10 - \square = 9</math>  <math>\square + 4 = 10</math>    <math>10 - 0 = \square</math> </p>	<p><b>Recognise small quantities</b></p>  <p><b>Count on</b></p>  <p>Count on, on number track in 1s.</p>	<p><b>Whole-part model</b></p>   <p>Fill in the missing numbers</p> <p><b>Balance image for concept of equality.</b></p> 	<p><b>Base 10</b></p>   <p>+</p>  <p>3 + 5 + 7</p>	<p><b>Whole-part model</b></p>   <p>Fill in the missing numbers</p>	<p><b>Partition and recombine</b></p> <p>Record partitioned steps in number sentences then add mentally.</p> <p> <math>40 + 20 = 60</math>  <math>6 + 7 = 13</math>  <math>60 + 13 = 73</math> </p> <p>Moving on to:</p> <p> <math>46 + 27 = 60 + 13 = 73</math> </p>  <p><b>Regrouping the 10.</b></p>
	<p><b>Hungarian frames</b></p>  <p>Use the pattern to complete the number sentences.</p>   <p>Use bonds of 10 to calculate bonds of 20.</p> <p>Doubles to 10 + 10.</p>	<p><b>Develop knowledge of fact families</b></p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p><math>10 = 3 + 7</math></p> <p><math>10 = 7 + 3</math></p> <p><math>10 - 7 = 3</math></p> </div>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p><math>20 = 3 + 17</math></p> <p><math>20 = 17 + 3</math></p> <p><math>20 - 3 = 17</math></p> </div>	<p><b>Balance image for concept of equality.</b></p>  <p> <math>9 = 9</math>  <math>9 = 8 + 1</math>  <math>9 = 7 + 2</math>  <math>8 + 1 = 7 + 2</math> </p>  <p> <math>10 = 10</math>  <math>10 = 8 + 2</math>  <math>10 = 6 + 4</math>  <math>8 + 2 = 6 + 4</math> </p>	<p><b>Adding more than two numbers</b></p>  <p>3 + 5 + 7</p> <p>Strategy to include looking for facts or bonds that are useful e.g. bonds up to and including 10, doubles or adding 10 to a given number.</p> <p><math>6 + 3 + 4 = 13</math></p>	<p><b>Adjustment strategy</b></p> <p> <math>5 + 9 =</math>  <math>5 + 10 - 1 = 14</math> </p>   <p>(Round and adjust)</p> <p><b>Doubles then near doubles</b></p>  <p> <math>8 + 9 = 8 + 8 + 1</math> </p> <p> <math>5 + 6 =</math>  <math>5 + 5 + 1 = 11</math> </p> <p> <math>7 + 8 =</math>  <math>8 + 8 - 1 = 15</math> </p>	<p><b>Balance in the equation</b></p> <p> <math>14 = 8 + 6,</math>  <math>7 + 6 = 8 + 5</math>  <math>\square = 13 + 9</math>  <math>3 + \square + 6 = 16</math>  <math>14 + \diamond = 15 + 27</math> </p> <p><b>Decision making</b></p> <p>Using statements such as:</p>

				$\overset{10}{6+3+4+7}+2=22$ Record thinking.	$47+50 =$ <b>Re-arranging</b> $18+4 =$ Tell me what you know about 4, e.g. $3+1$ , $2+2$ $18+4 =$ Rearrange the 4 into $2+2$ $18+2+2 = 20+2 = 22$  $59+24 =$ Partition the 24 into 20 +4 and rearrange the 4 into 1+3.  So $59+24 = 59+20+1+3 = 59+1+20+3 = 83$	Ben did $14 + 9 = 23$ How could he have done it?  Use sticks and smiles to partition and then smile to join key numbers together.  $\begin{array}{r} 28 + 45 \\ \hline 20 \quad 8 \quad 40 \quad 5 \end{array}$
Known facts	Represent & use number bonds and related subtraction facts within 20 Add and subtract 1 digit and 2 digit numbers to 20, including zero			Recall and use addition and subtraction facts to 20 fluently and derive and use related facts up to 100.		
Essential Knowledge	1 more	Number bonds: 5 and 6	10 more	Number bonds: 20, 12 and 13		
	Largest number first.	Number bonds: 7 and 8	Add 1 digit to 2 digit by bridging	Number bonds: 14 and 15		
	Add 10.	Number bonds: 9 and 10	Partition second number and add tens then ones.	Number bonds: 16 and 17		
	Ten plus ones.	Use number bonds of 10 to derive bonds of 11	Add 10 and multiples of 10.	Number bonds: 18 and 19		
	Doubles up to 10.		Doubles up to 20 and multiples of 5.	Partition and recombine.		
			Add near multiples of 10.			

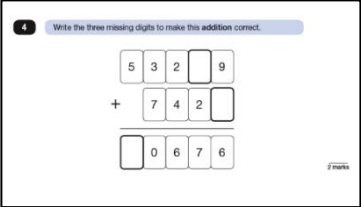
Year	3			4		
Layers of vocabulary  <b>Appendix Ia</b> Beck's Tiers of Vocabulary <b>Appendix Ib:</b> Vocabulary book	<b>Basic to subject specific (Beck's Tiers):</b> +, add, addition, more, plus make, sum, total altogether score double, near double one more, two more... ten more... one hundred more how many more to make...? how many more is... than...? how much more is...?			<b>Basic to subject specific (Beck's Tiers):</b> add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...?		
	<b>Instructional vocabulary:</b> explain your method explain how you got your answer give an example of... show how you... show your working			<b>Instructional vocabulary:</b> calculate, work out, solve investigate, question answer check		
NC 2014	Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction.			Add and subtract numbers with up to 4 digits using the formal written method of columnar addition and subtraction where appropriate. Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.		
Developing Declarative Procedural & Conditional Knowledge	<b>Near doubles</b> $13+14 =$ Double 13= 26 $26+1=27$ or Double 14 =28 $28-1=27$ <b>Using known facts</b>	<b>Start with least significant digit</b> $67$ $+ 24$ 11 (7+4) $+ 80$ (60+20) $91$	<b>Columnar addition</b> $625$ $+ 48$ $673$ 1 <b>Teach the carried digit.</b>	<b>Using known facts</b> $40 + 80 = 120$ using $4 + 8 = 12$ So $400 + 800 = 1200$ and $4000+8000=12,000$ <b>Remodelling strategy</b>	<b>Columnar addition</b> $587$ $+ 475$ $1062$ 11 "7 add 5 equals 12. That's 2 ones and 1 ten to carry over. 8 add 7 equals 15 and the ten to carry makes 16. That's 6 tens and 100 to carry over. 500 add 400 equals 900 and the 1	<b>Columnar addition (decimals) in contexts such as money and measurement</b> $12.45$ $7.36$ $+ 24.50$ $44.31$

	<p><math>40 + 80 = 120</math> using <math>4 + 8 = 12</math></p> <p>So <math>400 + 800 = 1200</math></p> <p><b>Remodelling strategy</b>  <math>243 + 198</math>  <math>241 + 200 = 441</math></p> <p><b>Place value materials to represent 3 digit numbers</b>  Base 10 and then place value counters.</p>  	<p>"7 add 4 equals 11 and 60 add 20 equals 80. 1 + 0 = 1 and 1 ten + 8 tens = 9 tens"</p> <p>"6 tens add 2 tens equals 8 tens"</p>  <p>625  <math>+ 48</math>  13 (5+8)  60 (20 +40)  + 600 (600 + 0)  <u>673</u></p> <p>All language in the context of the place value and added in columns, lining up the digits.</p> <p>Teaching point: no more than 9 in any given column following <b>regrouping</b>.</p>	<p><b>Representing problems</b></p> <p>There are 334 children at Springfield School and 75 at Oak Nursery. How many children are there altogether?</p>	<p><math>3548 + 1998</math></p> <p><math>3546 + 2000 = 5546</math></p> <p><b>Place value materials to represent calculations</b></p>	<p>hundred to carry makes 1000"</p> <p>7648  <math>+ 1486</math>  14 (8+6)  120 (40+80)  1000 (600+400)  + 8000 (7000+1000)  <u>9134</u></p> <p>7648  <math>+ 1486</math>  <u>9134</u>  111</p>	<p><b>Representing problems</b></p> <p><b>Conditional knowledge</b></p>  <p>Mr and Mrs Jones are saving for a holiday.  Mr Jones has saved £742.  Mrs Jones has saved £1509.  The holiday costs £2410.  How much more do they need to save?</p> 
<p>Known facts</p>	<p>Derive and use addition and subtraction facts to 100, e.g., <math>33 + 67 = 100</math>.</p>	<p>Derive and use addition and subtraction facts (for multiples of 10) to 1000, e.g., <math>330 + 670 = 1000</math>.</p>				

Essential knowledge	Add single digit bridging through boundaries	Add multiples of 10,100	Fluency of 2 digit + 2 digit	Add multiples of 10, 100 and 1000
	Partition second number to add	Pairs of 100 (complements of 100)	Partition second number to add	Decimal pairs of 10 and 1
	Use near doubles to add	Add near multiples of 10 and 100 by rounding and adjusting	Use near doubles to add	Adjust both numbers before adding
	Partition and recombine		Add near multiples	Partition and recombine


Year	5		6	
Layers of vocabulary  <b>Appendix Ia</b> Beck's Tiers of Vocabulary <b>Appendix Ib:</b> Vocabulary book	<b>Basic to subject specific (Beck's Tiers):</b> add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...?		<b>Basic to subject specific (Beck's Tiers):</b> add, addition, more, plus, increase sum, total, altogether score double, near double how many more to make...?	
	<b>Instructional vocabulary:</b> put, place arrange, rearrange change, change over split, separate		<b>Instructional vocabulary:</b> put, place arrange, rearrange change, change over adjusting, adjust split, separate carry on, continue, repeat what comes next? predict describe the pattern, describe the rule find, find all, find different investigate	
	NFER Arithmetic		NFER Arithmetic	
NC 2014	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction). Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.		Solve problems involving addition, subtraction, multiplication, and division.	
Developing Conceptual/ Procedural Understanding	<b>Columnar addition</b> Include calculations involving more than 2 numbers and carrying figures >1.	<b>Representing problems</b> <b>Procedural knowledge</b>	<b>Columnar addition</b> Include calculations with up to 3 'empty columns'. 128.7 + 3.014	<b>Representing problems</b> <b>Conditional knowledge</b>



	$\begin{array}{r} 25567 \\ 16397 \\ \hline \pm 15984 \\ 57948 \\ \hline 1121 \end{array}$ <p>Include calculations with 'empty columns'.</p> $124.9 + 7.25$ $\begin{array}{r} 124.90 \\ + 7.25 \\ \hline 132.25 \\ \hline 11 \end{array}$	<p>If 2541 is the answer, what's the question? - Can you create three addition calculations? - Can you create three subtraction calculations? - Did you use a strategy?</p> 	$\begin{array}{r} 128.700 \\ + 3.014 \\ \hline 131.714 \\ \hline 1 \end{array}$	<p>7208 females attended a concert as well as 8963 males. There were originally 20000 seats on sale. How many empty seats were there at the concert?</p> <p>On Saturday, a museum has 50,285 visitors. On Sunday, the museum has 10,500 more visitors than Saturday. The curator says, "We have had over one hundred thousand visitors across the two days." Do you agree? Explain your answer</p>
Known facts	Derive and use addition and subtraction facts to 10 and 1, e.g. 3.3+ 6.7 =10 and so 0.33 + 0.67 = 1.		All the KS2 required facts	
Essential knowledge	Fluency of 2 digit + 2 digit including with decimals	Add multiples of 10, 100, 1000 and tenths	Fluency of 2 digit + 2 digit including with decimals	Add multiples of 10, 100, 1000, tenths and hundredths
	Partition second number to add	Use number facts, bridging and place value	Partition second number to add	Use number facts, bridging and place value
	Adjust numbers to add	Partition and recombine	Adjust numbers to add	Partition and recombine

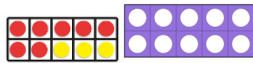
## Subtraction

	<p>Reception: ELG 2021</p> <ul style="list-style-type: none"> <li>Have an understanding of number to 10, linking names of numbers, numerals, their value, and their position in the counting order.</li> </ul>
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EYFS	<ul style="list-style-type: none"> <li>• Subitise (recognise quantities without counting) up to 5.</li> <li>• Automatically recall number bonds for numbers 0-5 and <i>for 10</i>, including corresponding partitioning facts.</li> <li>• Automatically recall double facts up 5+5</li> <li>• Compare sets of objects up to 10 in different contexts, considering size and difference.</li> <li>• Explore patterns of numbers within numbers up to 10, including evens and odds.</li> </ul>	
Year	1	2
<p>Layers of vocabulary</p>  <p>Appendix Ia Beck's Tiers of Vocabulary Appendix Ib: Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>take away, distance between, difference between, less than. How many more? How much greater? How many fewer? how much more is...? – subtract, take (away), minus, leave, how many are left/left over? how many have gone? one less, two less, ten less... how many fewer is... than...? how much less is...? difference between half, halve = equals, sign, is the same as</p> <p><b>Instructional vocabulary:</b></p> <p>start from, start with, start at look at point, to show</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>subtract, subtraction, take (away), minus leave, how many are left/left over? one less, two less... ten less... one hundred less how many fewer is... than...? how much less is...? difference between half, halve = equals, sign, is the same as tens boundary</p> <p>difference, partition, rearrange, inverse, place value</p> <p><b>Instructional vocabulary:</b></p> <p>tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of... show how you...</p>
NC 2014	Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs.	Using concrete objects and pictorial representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods
	Concrete, pictorial, abstract	Concrete, pictorial, abstract

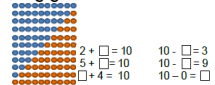
Developing Declarative, procedural, and conditional knowledge.

**Number bonds**



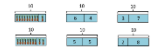
Ten Frames

Difference between 7 and 10.



$2 + \square = 10$      $10 - \square = 3$   
 $5 + \square = 10$      $10 - \square = 9$   
 $\square + 4 = 10$      $10 - 0 = \square$

Use the pattern to complete the number sentences.



6 less than 10 is 4.

Count out, then count how many are left. Remove from the set.

$7 - 4 = 3$



Count back on a number track.

$15 - 6 = 9$



Difference between.

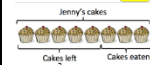
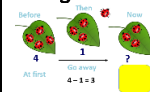
$13 - 8$

$= \underline{\quad}$



$8 + \underline{\quad} = 13$

**Subtraction-take away**



$8 - 3 = ?$

**Subtraction-finding the difference**

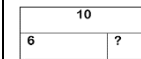


How many more cakes does Peter have than Jenny?  $8 - 3 = ?$

Develop knowledge of fact families.

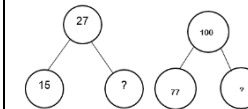
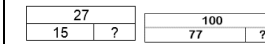


**Whole-part model**



Fill in the missing numbers

**Whole-part model**

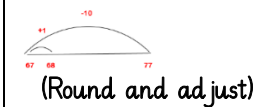


Fill in the missing numbers

All answers to be recorded in a number sentence following any informal recording.

**Adjustment strategy**

$77 - 9 =$   
 $77 - 10 + 1 = 67 + 1$   
 $= 68$



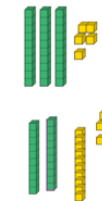
What is the nearest 10?

$55 - 27 =$   
 $55 - 30 + 3 = 25 + 3$   
 $= 28$

**Re-arranging**

$35 - 8 =$

Build 35 and then rearrange into 20 and 15



Tell me what you know about 8, e.g.  $2 + 6, 5 + 3$

$35 - 8 =$

Rearrange the 8 into  $5 + 3$

So  $35 - 5 - 3 = 30 - 3 = 27$

$55 - 27 =$

Subtract mentally pairs of multiples of 10 using known facts

$60 - 20 = 40$   
 because  $6 - 2 = 4$

**Partitioning of the second number strategy**

$74 - 47$

$74 - 40 = 34$

$34 - 4 - 3 = 27$

$74 - 47 =$

$77 - 50 = 27$

**Balance in the equation**

$35 - \square = 31$   
 $\square - 12 = 34$

$91 - 48 =$   
 Add 2 to both sides  
 $93 - 50 = 43$

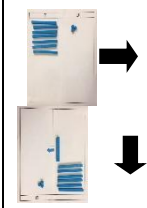


Partition the 27  
 into 20 + 7 and  
 rearrange the 7  
 into 5 + 2.  
 So  $55 - 27 = 55$   
 $-20 -5 -2$

$= 35 - 5 - 2$   
 $= 28$

**Taking away and exchanging**

$73 - 46 =$



What do we know  
 Exchange to make  
 about 76?  
 '60 and 13'.

$20 - \square = 14$   
 $- 3$   
 (Open-ended)  
 $18 - \square = 15$   
 $- \square$

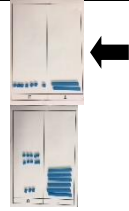
**Decision making**


$27 - \square = 12$


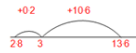

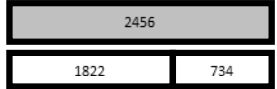
Sam works out

$27 - 15 = 12.$

How could he have done this?

				 <p>73 - 46 = 27 Now take away the 46.</p>	
Known facts	Represent & use number bonds and related subtraction facts within 20 Add and subtract 1 digit and 2 digit numbers to 20, including zero		Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100.		
Essential knowledge	1 less	Number bonds: subtraction 5 and 6	10 less	Number bonds: subtraction 20, 12 and 13	
	Count back	Number bonds: subtraction 7 and 8	Subtract 1 digit from 2 digit by bridging	Number bonds: subtraction 14 and 15	
	Subtract 10.	Number bonds: subtraction 9 and 10	Partition second number and count back in tens then ones.	Number bonds: subtraction 16 and 17	
	Teens subtract 10	Difference between	Subtract 10 and multiples of 10.	Number bonds: subtraction 18 and 19	
			Subtract near multiples of 10.	Difference between	
			Add near multiples of 10.		

Year	3			4		
<p>Layers of vocabulary</p>  <p><b>Appendix Ia</b> Beck's Tiers of Vocabulary</p> <p><b>Appendix Ib:</b> Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>subtract, subtraction, take (away), minus leave, how many are left/left over? one less, two less... ten less... one hundred less how many fewer is... than...? how much less is...? difference between half, halve = equals, sign, is the same as tens boundary, hundreds boundary</p> <p>exchange, carried digits</p> <p><b>Instructional vocabulary:</b></p> <p>explain your method explain how you got your answer give an example of... show how you... show your working</p> <p>NFER Arithmetic</p>			<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? difference between half, halve how many more/fewer is... than...? how much more/less is...? equals, sign, is the same as tens boundary, hundreds boundary, inverse</p> <p>exchange, carried digits</p> <p><b>Instructional vocabulary:</b></p> <p>calculate, work out, solve investigate, question answer check</p> <p>NFER Arithmetic</p>		
NC 2014	Add and subtract numbers with up to 3 digits, using formal written methods of columnar addition and subtraction. Least significant digit is always dealt with first to establish if the exchange is needed.			Add and subtract numbers with up to 4 digits using the formal written method of columnar addition and subtraction where appropriate. Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.		
Developing declarative, procedural, and	<b>Subtract mentally pairs of multiples of 100 using known facts</b>	<b>Start with least significant digit - decomposition</b>	<b>Columnar subtraction</b>	<b>Subtract mentally pairs of multiples of 1000 using known facts</b>	<b>Columnar subtraction</b> 2344 - 187	<b>Representing problems</b> Check the answer to the following calculations using
		81 = 80 + 1				

<p>conditional knowledge</p>	<p><math>600 - 200 = 400</math> because <math>6 - 2 = 4</math></p> <p>Remodelling strategy (keeping the difference the same) <math>502 - 198</math></p> <p><math>504 - 200 = 304</math></p> <p>Re-arranging</p> <p>Use of apparatus to understand rearrangements, e.g. 55 as 40 and 15 (not as part of calculations).</p> <p>Place value materials to represent numbers in calculations</p> 	<p><math>50 \begin{array}{r} - 57 \\ \hline \end{array}</math></p> <p>24</p> <p>81 = 70</p> <p>11</p> <p><math>50 \begin{array}{r} - 57 \\ \hline \end{array}</math></p> <p><math>20 \begin{array}{r} - 24 \\ \hline \end{array}</math></p> <p>754    700    50</p> <p>4</p>	<p><math>\begin{array}{r} 6 \ 14 \ 1 \\ 784 \\ - 286 \\ \hline 468 \end{array}</math></p> <p>Emphasis on language of number facts, i.e., 14 subtract 6, 14 subtract 8, and 6 subtract 2</p> <p>Representing problems</p> <p>There are 386 pupils at Oak Primary. If 79</p>	<p><math>6000 - 2000 = 4000</math> because</p> <p><math>6 - 2 = 4</math></p> <p>Remodelling strategy (keeping the difference the same) <math>3548 - 1998</math></p> <p><math>3550 - 2000 = 1550</math></p> <p>Find the difference strategy <math>13.6 - 2.8 =</math></p>  <p><math>13.6 - 2.8 = 10.8</math></p>	<p><math>\begin{array}{r} 2 \ 13 \ 1 \\ 2344 \\ - 187 \\ \hline 2157 \end{array}</math></p> <p><math>64.67 - 2684</math></p> <p><math>\begin{array}{r} 5 \ 13 \ 1 \\ 6467 \\ - 2684 \\ \hline 3783 \end{array}</math></p> <p>Columnar subtraction (decimals) in contexts such as money and measurement</p> <p><math>32.34 - 14.18</math></p> <p><math>\begin{array}{r} 2 \ 1 \ 2 \ 1 \\ 32.34 \\ - 14.18 \\ \hline 18.16 \end{array}</math></p>	<p>the inverse. Show all your working.</p>  <p>Conditional knowledge</p> <p><math>2456 - 734 = 1822</math></p> 
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$$\begin{array}{r} - 86 \\ 80 \quad \underline{\quad} 6 \\ - \quad \underline{\quad} \end{array}$$

$$754 \quad 600 \quad 140$$

$$14$$

$$\begin{array}{r} - 86 \\ 80 \quad \underline{\quad} 6 \end{array}$$

$$\begin{array}{r} 668 \quad \underline{600} \\ 60 \quad \underline{\quad} 8 \end{array}$$


"It's tricky to take 6 from 4 and 80 from 50. I need to rearrange the number. I will exchange one ten from 50 which leaves 40 and makes 14 in the units. 40 to subtract 80 is tricky. I will exchange one hundred from 700 and make 140. 14 subtract 6 equals 8. 140 subtract 80 equals 60 and 600

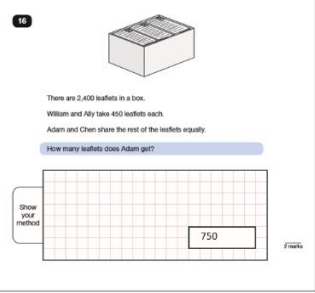
pupils have sandwiches, how many have dinners?

386	
?	79



		subtract 0 equals 600."			
Known facts	Derive and use addition and subtraction facts to 100, e.g. $33 + 67 = 100$ .		Derive and use addition and subtraction facts (for multiples of 10) to 1000, e.g. $330 + 670 = 1000$ .		
Essential knowledge	Subtract single digit bridging through boundaries	Subtract multiples of 10, 100	Fluency of 2 digit - 2 digit	Subtract multiples of 10, 100 and 1000	
	Partition second number to subtract	Pairs of 100 (complements of 100)	Partition second number to subtract	Decimal subtraction from 10 or 1	
	Difference between	Subtract near multiples of 10 and 100 by rounding and adjusting	Difference between	Subtract near multiples by rounding and adjusting	
	Partition and recombine				

Year	5	6
<p>Layers of vocabulary</p>  <p>Appendix 2a Beck's Tiers of Vocabulary Appendix 2b: Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>subtract, subtraction, take (away), minus, leave, how many are left/left over? ten less... one hundred less how many fewer is... than...? how much less is...? difference between half, halve = equals, sign, is the same as tens boundary, hundreds boundary, inverse, <b>units boundary</b>, tenths boundary</p> <p>exchange, carried digits</p> <p><b>Instructional vocabulary:</b></p> <p>put, place arrange, rearrange change, change over adjusting, adjust split, separate</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>subtract, subtraction, take (away), minus, decrease leave, how many are left/left over? difference between half, halve how many more/fewer is... than...? how much more/less is...? equals, sign, is the same as tens boundary, hundreds boundary, <b>units boundary</b>, tenths boundary, inverse</p> <p><b>Instructional vocabulary:</b></p> <p>put, place arrange, rearrange change, change over adjusting, adjust split, separate</p> <p>carry on, continue, repeat what comes next? predict describe the pattern, describe the rule</p>



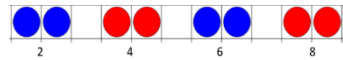
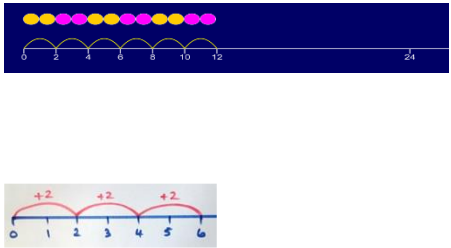
	NFER Arithmetic	find, find all, find different investigate	NFER Arithmetic	
NC 2014	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction). Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.	Solve problems involving addition, subtraction, multiplication and division.		
Developing declarative, procedural, and conditional knowledge	<p><b>Columnar subtraction</b></p> $\begin{array}{r} & 2 & 3 & 1 & & \\ & 5 & 2 & 3 & 4 & 4 \\ - & 1 & 1 & 8 & 7 & \\ \hline & 5 & 1 & 1 & 5 & 7 \end{array}$ <p>Include calculations with 'empty columns'. <math>324.9 - 7.25</math></p> $\begin{array}{r} 1181 \\ 324.90 \\ - 7.25 \\ \hline 317.65 \end{array}$	<p><b>Representing problems</b></p> <p>Kangchenjunga is the third highest mountain in the world at 28,169 feet above sea level. Lhotse is the fourth highest at 27,960 feet above sea level. Find the difference in heights mentally.</p> <p>Keeping the difference, the same to make the numbers easier to calculate with.</p> <p><math>122,456 - 11,999</math></p>	<p><b>Columnar subtraction</b></p> <p>Include calculations with up to 3 'empty columns'. <math>128.7 - 3.014</math></p> $\begin{array}{r} & 6 & 9 & 11 & & \\ & 128.700 \\ - & 3.014 \\ \hline & 125.686 \end{array}$	<p><b>Representing problems</b></p> <p>Katie was given the calculation below <math>47326 - 1900 =</math> She said "I will just take off 2000 then subtract another 100 so my answer is 45126." Is she correct? Would you use her method? Explain your answer</p> 

		$122,457 - 12,000$		<div style="border: 1px solid black; padding: 5px; margin-bottom: 5px;"> <div style="text-align: center; border-bottom: 1px solid black;">2,400</div> <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 25%;">450</td> <td style="width: 25%;">450</td> <td style="width: 25%;">?</td> <td style="width: 25%;"></td> </tr> </table> </div> <p>Conditional knowledge</p> <div style="border: 1px solid black; padding: 5px; margin-top: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%; text-align: center; vertical-align: middle;"><b>14</b></td> <td style="width: 70%; border: 1px solid black; text-align: center;">= 5,776 - 855</td> <td style="width: 20%;"></td> </tr> </table> </div>	450	450	?		<b>14</b>	= 5,776 - 855	
450	450	?									
<b>14</b>	= 5,776 - 855										
Known facts	Derive and use addition and subtraction facts to 10 and 1, e.g. $3.3 + 6.7 = 10$ leads to $10 - 3.3 = 6.7$ and $0.33 + 0.67 = 1$ so $1 - 0.67 = 0.33$		All the KS2 required facts								
Essential knowledge	Fluency of 2 digit - 2 digit including with decimals	Subtract multiples of 10, 100, 1000 and tenths	Fluency of 2 digit - 2 digit including with decimals	Subtract multiples of 10, 100, 1000, tenths and hundredths							
	Partition second number to subtract	Use number facts, bridging and place value	Partition second number to subtract	Use number facts, bridging and place value							
	Adjust numbers to subtract	Difference between	Adjust numbers to subtract	Difference between							


## Multiplication

EYFS	<p><b>Reception: ELG 2021</b></p> <ul style="list-style-type: none"> <li>• Have an understanding of number to 10, linking names of numbers, numerals, their value, and their position in the counting order.</li> <li>• Subitise (recognise quantities without counting) up to 5.</li> <li>• Automatically recall number bonds for numbers 0-5 and <i>for 10</i>, including corresponding partitioning facts.</li> <li>• Automatically recall double facts up to 5+5</li> <li>• Compare sets of objects up to 10 in different contexts, considering size and difference</li> </ul>
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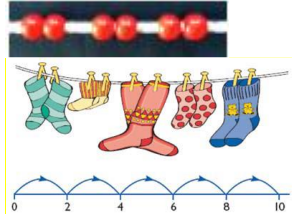

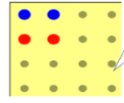
- Explore patterns of numbers within numbers up to 10, including evens and odds

Year	1	2		
Layers of vocabulary  <b>Appendix Ia</b> Beck's Tiers of Vocabulary <b>Appendix Ib:</b> Vocabulary book	<b>Basic to subject specific (Beck's Tiers):</b> count in ones, twos... tens... array, groups of, equal groups odd, even  <b>Instructional vocabulary:</b> carry on, continue repeat what comes next? find, choose, collect use, make, build tell me, describe, pick out, talk about, explain, show me, read, write, record	<b>Basic to subject specific (Beck's Tiers):</b> lots of, groups of $\times$ , times, multiply, multiplied by multiple of once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally  <b>Instructional vocabulary:</b> carry on, continue, repeat, what comes next? predict describe the pattern describe the rule find, find all, find different, investigate		
NC 2014	Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals (=) signs.		
Developing declarative, procedural, and conditional knowledge	<b>Grouping</b>  2 frogs on each lily pad  	<b>Arrays</b> (rectangular arrangements to show equal groups)	<b>Repeated addition and skip counting</b> 	<b>Commutativity</b>

There are 5 hedgehogs in each group. How many hedgehogs are there altogether?

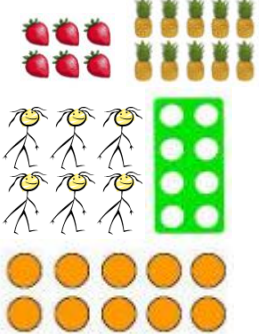



**Doubles**

Use your peg board to show  
4 groups of 2  
3 groups of 10  
5 groups of 5


Is it true that all groups of 2 are even?  
Do all groups of 10 end in 0?

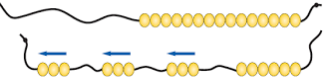
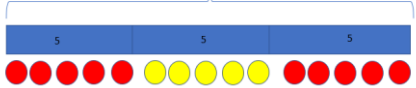
6 can be put into groups of 2.  
 $2 + 2 + 2 = 6$

10 can be put into groups of 2 and 5.  
 $2 + 2 + 2 + 2 = 10$   
 $5 + 5 = 10$


**Grouping**



5 frogs on each lily pad  $5 \times 3 = 15$

**Building tables**




Build tables using counting stick-forwards and backwards and with missing jumps using doubling and halving.


Introduce the x symbol once repeated addition is understood.

Complete  
6, 8, 10, .....20  
15, 20, 25.....50  
60, 70, 80.....100


**Grouping**



$4 \times 2 = 8$   
 $2 \times 4 = 8$



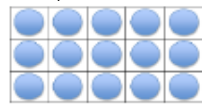
$4 \times 2 = 8$   
 $2 \times 4 = 8$




$5 + 5 + 5 + 5 = 20$   
 $5 \times 4 = 20$   
4 multiplied by 5  
4 groups of 5

**Conditional Knowledge**

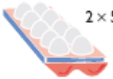
How many number sentences can you write to describe this array? Can you use addition, multiplication and division?



Explain your answers.




$5 \times 2 = 10$   
 $2 \times 5 = 10$



$2 \times 5 = 10$   
 $5 \times 2 = 10$

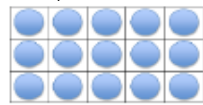
$5 \times 2 = 2 \times 5$



$5 + 5 + 5 + 5 + 5 = 30$   
 $5 \times 6 = 30$   
5 multiplied by 6  
6 groups of 5  
6 hops of 5


**Conditional Knowledge**

How many number sentences can you write to describe this array? Can you use addition, multiplication and division?


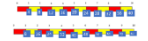

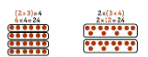
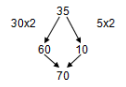

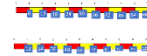

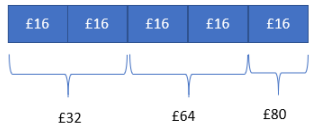


Explain your answers.

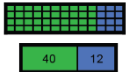
				6. Write a story to go with this equation. $6 \times 10 = 60$ 7. Complete the calculations. $7 \times 5 = \square$ $10 \times 4 = \square$ $9 \times 2 = \square$
Known facts	Count in multiples of twos, fives and tens.		Recall and use $\times$ and $\div$ facts for the 2, 5 and 10 $\times$ tables, including recognising odd and even numbers.	
Essential Knowledge	Count in 2s	Doubles up to 10	2 $\times$ table	Doubles up to 20
	Count in 10s	Double multiples of 10	10 $\times$ table	Doubles of multiples of 5
	Count in 5s	Count in 2s, 5s and 10s	5 $\times$ table	Count in 3s


Year	3	4
Layers of vocabulary  <b>Appendix Ia</b> Beck's Tiers of Vocabulary <b>Appendix Ib:</b> Vocabulary book	<b>Basic to subject specific (Beck's Tiers):</b> lots of, groups of $\times$ , times, multiply, multiplication, multiplied by multiple of, product once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally one each, two each, three each...  <b>Instructional vocabulary:</b> carry on, continue repeat what comes next? predict describe the pattern, describe the rule find, find all, find different, investigate	<b>Basic to subject specific (Beck's Tiers):</b> lots of, groups of times, multiply, multiplication, multiplied by multiple of, product once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve, factor, multiple  <b>Instructional vocabulary:</b> carry on, continue, repeat what comes next? predict describe the pattern, describe the rule pattern, puzzle, calculate, calculation, mental calculation, method, jotting, answer right, correct, wrong what could we try next? how did you work it out? number sentence sign, operation, symbol, equation



	choose, decide, collect	
NC 2014	Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including 2 digit numbers times 1 digit numbers progressing to formal written methods.	Multiply 2 digit and 3 digit numbers by a 1 digit number using formal written layout. Solve problems involving multiplying and adding.

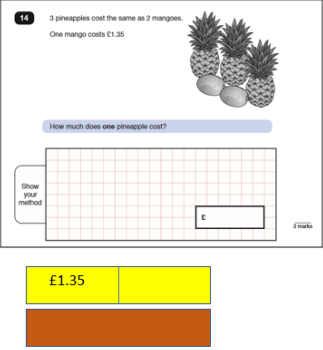
<p>Developing declarative, procedural, conditional knowledge</p>	<p><b>Building tables</b></p>  <p>For example, build tables using counting stick- forwards and backwards with doubling and halving</p>   <p><b>Using known facts</b></p> <p>If <math>3 \times 2 = 6</math>, then  <math>30 \times 2 = 60</math>, <math>60 \div 3 = 20</math> and  <math>30 = 60 \div 2</math>.</p> <p><b>Associativity</b></p> <p><math>(2 \times 3) \times 4 = 2 \times (3 \times 4)</math></p> 	<p><b>Partitioning strategy to double</b></p> <p>Double 35</p>  <p><b>Place value materials to represent calculations</b></p> <p><b>Partitioning</b>      Informal recording of partitioned numbers</p> <p><math>15 \times 5 = 75</math></p> <p><math>10 \times 5 = 50</math></p> <p><math>5 \times 5 = 25</math></p> <p><math>27 \times 3 = 81</math></p> <p><math>20 \times 3 = 60</math></p> <p><math>7 \times 3 = 21</math></p>	<p><b>Grid method</b></p> <p><math>23 \times 8 =</math></p> <p><math>20 \times 8 = 160</math></p> <p><math>3 \times 8 = 24</math></p> <p><math>23 \times 8 = 184</math></p> <table border="1" data-bbox="929 502 1041 550"> <tr><td>x</td><td>20</td><td>3</td></tr> <tr><td>8</td><td></td><td></td></tr> </table> <p><b>Short multiplication</b></p> <p>Expanded</p> <p><math>23</math></p> <p><math>\times 8</math></p> <p><math>24</math> (8 x 3)</p> <p><math>160</math> (8 x 20)</p> <p><math>184</math></p> <p>leading to compact</p> <p><math>23</math></p> <p><math>\times 8</math></p>	x	20	3	8			<p><b>Building tables</b></p>  <p>For example, build tables using counting stick- forwards and backwards with doubling and halving</p>  <p><b>Using known facts</b></p> <p>If <math>2 \times 3 = 6</math> then <math>200 \times 3 = 600</math> and <math>600 \div 3 = 200</math></p> <p><b>Distributivity</b></p> <p><math>3 \times (2 + 4) = 3 \times 2 + 3 \times 4</math></p> <p>So the '3' can be 'distributed' across the '2 + 4' into 3 times 2 and 3 times 4</p>  <p>leading to</p> <p><math>13 \times 4 = 10 \times 4 + 3 \times 4 = 52</math></p>	<p><b>Place value materials to represent calculations</b></p> <p><b>Grid method</b> (if needed for conceptual understanding)</p> <p><math>346 \times 9</math></p> <table border="1" data-bbox="1444 566 1568 606"> <tr><td>x</td><td>300</td><td>40</td><td>6</td></tr> <tr><td>9</td><td></td><td></td><td></td></tr> </table> <p><b>Short multiplication</b></p> <p>Expanded</p> <p><math>346</math></p> <p><math>\times 9</math></p> <p><math>54</math> (9 x 6)</p> <p><math>360</math> (9 x 40)</p> <p><math>2700</math> (9 x 300)</p> <p><math>3114</math></p> <p>leading to compact</p> <p><math>346</math></p>	x	300	40	6	9				<p><b>Representing problems</b></p> <p><b>Conditional Knowledge</b></p> <p>Multiply a number by itself and then make one factor one more and the other one less. What do you notice? Does this always happen?</p> <p>Eg <math>4 \times 4 = 16</math>      <math>6 \times 6 = 36</math></p> <p><math>5 \times 3 = 15</math>      <math>7 \times 5 = 35</math></p> <p>Try out more examples to prove your thinking.</p> <div data-bbox="1713 798 2027 933"> <p>2 A group of friends earns £80 by washing cars. They share the money equally. They get £16 each. How many friends are in the group?</p> <p><input type="text"/> mark</p> </div>  <p><b>Place &lt;, &gt;, or = in these number sentences to make them correct.</b></p> <p><math>50 \times 4</math> <input type="checkbox"/> <math>4 \times 50</math></p> <p><math>4 \times 50</math> <input type="checkbox"/> <math>40 \times 5</math></p> <p><math>200 \times 5</math> <input type="checkbox"/> <math>3 \times 300</math></p>
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8																				
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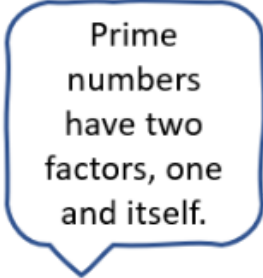


		<p>"20 multiplied by 3 equals 60 and 7 multiplied by 3 equals 21. 60 add 21 equals 81."</p>	<p><del>18</del> 2</p> <p><b>Representing problems</b></p> <p>A group of aliens live on Planet Xert. Tinions have three legs, Quinions have four legs. The group has 22 legs altogether. How many Tinions and Quinions might there be? Is there more than one solution?</p>		$\begin{array}{r} \phantom{0}x\phantom{0}9 \\ 311\cancel{4} \\ \phantom{0}4\phantom{0}5 \end{array}$	
Known facts	Recall and use $\times$ and $\div$ facts for the 3, 4 and 8 $\times$ tables		Recall $\times$ and $\div$ facts for $\times$ tables up to $12 \times 12$ .			
Essential knowledge	Review 2x, 5x and 10x	Double 2 digit numbers	4x and 8x tables		10x bigger, 100 x bigger	
	4x table	3x table	3x, 6x and 12x tables		Double larger numbers and decimals	
	8 x table	6x table	3x and 9x tables		11x and 7x tables	

Year	5	6
<p>Layers of vocabulary</p>  <p>Appendix Ia Beck's Tiers of Vocabulary</p> <p>Appendix Ib: Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>lots of, groups of times, multiply, multiplication, multiplied by multiple of, product once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally</p> <p>factor, multiple, prime, composite</p> <p><b>Instructional vocabulary:</b></p> <p>carry on, continue, repeat what comes next? predict describe the pattern, describe the rule</p> <p>find, find all, find different investigate</p> <p>NFER Arithmetic</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>lots of, groups of times, multiply, multiplication, multiplied by multiple of, product once, twice, three times... ten times... times as (big, long, wide... and so on) repeated addition array row, column double, halve share, share equally</p> <p>factor, multiple, prime, composite</p> <p><b>Instructional vocabulary:</b></p> <p>carry on, continue, repeat what comes next? predict describe the pattern, describe the rule</p> <p>find, find all, find different investigate</p> <p>NFER Arithmetic</p>
<p>NC 2014</p>	<p>Multiply numbers up to 4 digits by a 1 or 2 digit number using a formal written method, including long multiplication for 2 digit numbers</p> <p>Solve problems involving multiplication and division including using knowledge of factors and multiples, squares and cubes</p> <p>Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign</p>	<p>Multiply multi-digit numbers up to 4 digits by a 2 digit whole number using the formal written method of long multiplication.</p> <p>Solve problems involving addition, subtraction, multiplication and division.</p>


		Solve problems involving multiplication and division including scaling by simple fractions and problems involving simple rates												
Developing declarative, procedural, conditional knowledge	<p><b>Building tables</b></p>  <p>For example, apply tables knowledge to multiples of 10, 100 and 1000 using counting stick- forwards and backwards with doubling and halving.</p>	<p><b>Grid method</b> (if needed for conceptual understanding)</p> $28 \times 27$ <table border="1" data-bbox="622 466 712 513"> <tr><td>x</td><td>20</td><td>8</td></tr> <tr><td>20</td><td></td><td></td></tr> <tr><td>7</td><td></td><td></td></tr> </table> <p>Addition to be done mentally or across followed by column addition</p>	x	20	8	20			7			<p>leading to compact</p> $\begin{array}{r} 28 \\ \times 27 \\ \hline 196 \\ 5 \\ \hline 560 \\ 1 \\ \hline 756 \\ 1 \end{array}$	<p><b>Building tables</b></p>  <p>For example, apply tables knowledge to decimals using counting stick- forwards and backwards with doubling and halving.</p>	<p>If place value is secure, use grid method for decimal multiplication</p> $0.75 \times 6$
	x	20	8											
20														
7														
<p><b>Using known facts</b></p> <p>If <math>2 \times 3 = 6</math> then <math>2000 \times 3 = 6000</math> and <math>200 \times 30 = 6000</math></p> <p><b>Place value materials to represent calculations</b></p>	<p><b>Long multiplication</b></p> <p>Expanded</p> $\begin{array}{r} 28 \\ \times 27 \\ \hline 56 \text{ (7} \times 8) \\ 140 \text{ (7} \times 20) \\ \hline 160 \text{ (20} \times 8) \\ 400 \text{ (20} \times 20) \\ \hline 756 \end{array}$	<p>“Place a zero to hold the ones, as everything is ten times bigger.”</p> <p>Extend to HTO x TO or ThHTO x TO as appropriate</p> <p><b>Representing problems</b></p>	<p><b>Using known facts</b></p> <p>If <math>2 \times 3 = 6</math> then <math>0.2 \times 3 = 0.6</math> and <math>0.02 \times 3 = 0.06</math></p> <p><b>Long multiplication</b></p> <p>Use expanded method first if needed to build conceptual understanding</p> $\begin{array}{r} 5172 \\ \times 27 \\ \hline 36204 \\ 151 \\ \hline 103440 \\ 1 \\ \hline 139644 \end{array}$	<p><math>0.7 \times 6 = 4.2</math></p> <p><math>0.05 \times 6 = 0.3</math></p> <p><math>0.75 \times 6 = 4.5</math></p> <p><b>Make explicit links between decimals and money</b></p> <table border="1" data-bbox="1697 906 1832 954"> <tr><td>x</td><td>0.7</td><td>0.05</td></tr> <tr><td>6</td><td></td><td></td></tr> </table> <p><b>Representing problems and conditional knowledge</b></p> <p>Amy is given the calculation <math>5413 \times 600</math>. She says “I can do this without a written method.” Write down the mental steps you think Amy could do.</p>	x	0.7	0.05	6						
x	0.7	0.05												
6														

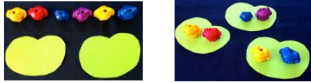
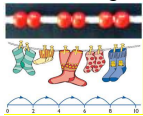
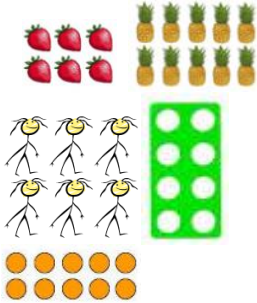





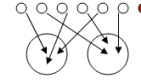



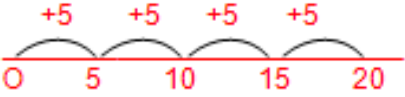
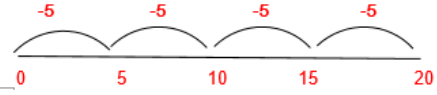

	<p><b>Short multiplication</b></p> <p>Use expanded method first if needed to build conceptual understanding</p> $\begin{array}{r} 4346 \\ \times 8 \\ \hline 34768 \\ 234 \end{array}$		<p>40 cupcakes cost £3.60, how much do 20 cupcakes cost? How much do 80 cupcakes cost? How much do 10 cupcakes cost?</p>		 <p>3 pineapples cost the same as 2 mangoes. One mango costs £1.35</p> <p>How much does one pineapple cost?</p> <p>Show your method</p> <p>£</p> <p>£1.35</p>
<p>Known facts</p>	<p>Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers</p> <p>Recall prime numbers up to 19</p> <p>Recognise and use square and cube numbers and the notation for squared (<sup>2</sup>) and cubed (<sup>3</sup>)</p>	<p>Identify common factors, common multiples and prime numbers</p>			

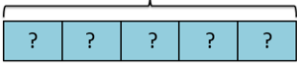
			<table border="1"> <tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr> <tr><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td><td>16</td><td>17</td><td>18</td><td>19</td><td>20</td></tr> <tr><td>21</td><td>22</td><td>23</td><td>24</td><td>25</td><td>26</td><td>27</td><td>28</td><td>29</td><td>30</td></tr> <tr><td>31</td><td>32</td><td>33</td><td>34</td><td>35</td><td>36</td><td>37</td><td>38</td><td>39</td><td>40</td></tr> <tr><td>41</td><td>42</td><td>43</td><td>44</td><td>45</td><td>46</td><td>47</td><td>48</td><td>49</td><td>50</td></tr> <tr><td>51</td><td>52</td><td>53</td><td>54</td><td>55</td><td>56</td><td>57</td><td>58</td><td>59</td><td>60</td></tr> <tr><td>61</td><td>62</td><td>63</td><td>64</td><td>65</td><td>66</td><td>67</td><td>68</td><td>69</td><td>70</td></tr> <tr><td>71</td><td>72</td><td>73</td><td>74</td><td>75</td><td>76</td><td>77</td><td>78</td><td>79</td><td>80</td></tr> <tr><td>81</td><td>82</td><td>83</td><td>84</td><td>85</td><td>86</td><td>87</td><td>88</td><td>89</td><td>90</td></tr> <tr><td>91</td><td>92</td><td>93</td><td>94</td><td>95</td><td>96</td><td>97</td><td>98</td><td>99</td><td>100</td></tr> </table>	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	
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Essential knowledge	4x and 8x tables	100, 1000 times bigger	Multiplication facts up to 12 x 12	Partition to multiply mentally																																																																																																				
	3x, 6x and 12x tables; 3x and 9x tables	10, 100, 1000 times smaller		Apply place value to derive multiplication facts, e.g. $3 \times 4 = 12$ so $3 \times 0.4 = 1.2$	Double larger numbers and decimals																																																																																																			
	11x and 7x tables	Double larger numbers and decimals			10 x smaller 100 x smaller																																																																																																			

## Division


EYFS	<p>Reception: ELG 2021</p> <ul style="list-style-type: none"> <li>• Have an understanding of number to 10, linking names of numbers, numerals, their value, and their position in the counting order.</li> <li>• Subitise (recognise quantities without counting) up to 5.</li> <li>• Automatically recall number bonds for numbers 0-5 and <i>for 10</i>, including corresponding partitioning facts.</li> <li>• Automatically recall double facts up 5+5</li> </ul>
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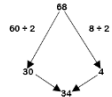


	<ul style="list-style-type: none"> <li>Compare sets of objects up to 10 in different contexts, considering size and difference.</li> <li>Explore patterns of numbers within numbers up to 10, including evens and odds.</li> </ul>	
Year	1	2
<p>Layers of vocabulary</p>  <p>Appendix Ia Beck's Tiers of Vocabulary</p> <p>Appendix Ib: Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>count in ones, twos... tens...</p> <p>share, groups of, equal groups,</p> <p>odd, even</p> <p><b>Instructional vocabulary:</b></p> <p>count out, share out, left, left over.</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>share, share equally one each, two each, three each... group in pairs, threes... tens equal groups of <math>\div</math>, divide, divided by, divided into left, left over.</p> <p><b>Instructional vocabulary:</b></p> <p>tell me, describe, name, pick out, discuss, talk about, explain, explain your method, explain how you got your answer, give an example of... show how you</p>
NC 2014	solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations, and arrays with the support of the teacher.	Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals (=) signs.


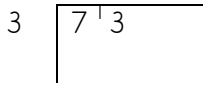
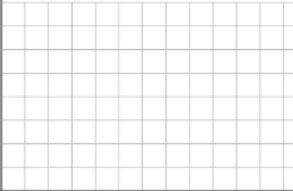
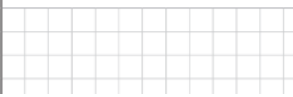
	Concrete, pictorial, abstract	Concrete, pictorial, abstract		
Developing declarative, procedural, conditional knowledge	<p><b>Grouping/Sharing models</b> Using practical contexts and cross-curricular links (PE) such as socks and shoes; animals in the ark to get into groups. Sharing models such as sharing pieces of fruit.</p> <p>Sharing into equal groups 6 frogs shared equally between 2 lily pads gives 3 frogs on each lily pad or Grouping in equal groups 6 frogs grouped in 2s need 3 lily pads to sit on</p>  <p>How many twos?</p> 	<p><b>Arrays</b> (rectangular arrangements to show equal groups)</p>  <p><b>Decision making</b></p> <p>How many cars can you make if you have 8 wheels?</p>  <p>How many different ways can you arrange 12 buttons in equal groups?</p> 	<p><b>Grouping/Sharing models</b> Introduce the <math>\div</math> symbol</p>  <p>15 frogs shared equally between three lily pads <math>15 \div 3 = 5</math> or 15 frogs grouped in 5s need 3 lily pads to sit on <math>15 \div 5 = 3</math></p> <p><math>15 \div 3 = 5</math> groups of 3 (grouping)</p>  <p><math>20 \div 2 = 10</math></p>  <p>There are 7 cakes and 2 children. How many cakes will they get each? (Leftovers/reminders introduced)</p>  <p><math>7 \div 2 = 3r1</math></p>	<p><b>Arrays representing the dividend</b></p>  <p><math>10 \div 2 = 5</math>  and <math>10 \div 5 = 2</math></p> <p><b>Repeated addition (to reach a given target)</b></p>  <p>There are 20 sweets in a bag. How many children can have 5 each?</p>  <p><b>Repeated subtraction (from a given quantity)</b></p>  <p><b>Links to tables</b></p>  <p>Use language of division linked to tables using counting stick</p> <p><b>Representing problems</b></p>


				<p>Jane has 30 cakes. She wants to share them equally between 5 boxes. How many cakes should go in each box?</p> <p style="text-align: center;">30</p>  <p style="text-align: right;"><math>30 \div 5 = 6</math></p> <p>Number of cakes in each box = 6</p>
Known facts	Count in multiples of twos, fives and tens.		Recall and use $\times$ and $\div$ facts for the 2, 5 and 10 $\times$ tables, including recognising odd and even numbers.	
Essential Knowledge	Count back in 2s	Halves up to 10	Division facts (2 $\times$ table)	Halves up to 20
	Count back in 10s	Halve multiples of 10	Division facts (10 $\times$ table)	Review division facts (2 $\times$ , 5 $\times$ , 10 $\times$ tables)
	Count back in 5s	How many 2s? 5s? 10s?	Division facts (5 $\times$ table)	Count back in 3s
Tests of divisibility	All even numbers will divide by 2		All numbers ending in 0 will divide by 10	All numbers ending in 5 and 0 will divide by 5



Year	3	4
<p>Layers of vocabulary</p>  <p>Appendix Ia Beck's Tiers of Vocabulary Appendix Ib: Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>share, share equally one each, two each, three each...</p> <p>group in pairs, threes... tens equal groups of <math>\div</math>, divide, division, divided by, divided into left, left over, remainder, dividend, divisor</p> <p><b>Instructional vocabulary:</b></p> <p>calculate, work out, solve, investigate question, answer, check</p>	<p><b>Basic to subject specific (Beck's Tiers):</b></p> <p>share, share equally one each, two each, three each...</p> <p>group in pairs, threes... tens equal groups of <math>\div</math>, divide, division, divided by, divided into left, left over, remainder, dividend, divisor</p> <p><b>Instructional vocabulary:</b></p> <p>calculate, work out, solve, investigate, question, answer, check</p>
<p>NC 2014</p>	<p>Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including 2 digit numbers times 1 digit numbers progressing to formal written methods.</p>	<p>Practise to become fluent in the formal written method of short division with exact answers.</p>

<p>Developing declarative, procedural, conditional knowledge</p>	<p><b>Links to tables</b></p>  <p>For example, use language of division linked to tables using counting stick</p> <p><b>Using known facts</b></p> <p>If <math>3 \times 2 = 6</math>, then <math>30 \times 2 = 60</math>, <math>60 \div 3 = 20</math> and <math>30 = 60 \div 2</math>.</p> <p><b>Partitioning strategy to halve</b></p> <p>Halve 68</p>  <p><b>Rearranging the dividend to find multiples of the divisor.</b></p> <p><math>48 \div 3 =</math></p> <p>'What do I know about the 3 x tables?'</p> <p>"I know <math>3 \times 10 = 30</math> and <math>3 \times 6 = 18</math>."</p>	<p><b>Place value materials to represent calculations</b></p> <p>Introduce the 'bus stop' bracket and vinculum notation.</p> <p><b>Short division ( <math>72 = 60 + 12</math> )</b></p> $72 \div 3 =$ $\begin{array}{r} 24 \\ 3 \overline{) 72} \end{array}$ <p>'72 divided by 3. 7 tens shared equally between 3 is 2 with a remainder of 1 ten. Exchange the 1 ten for 10 ones. I now have 12 ones which shared equally between 3 is 4. The answer is 24.'</p> <p><b>Representing problems and conditional knowledge</b></p>	<p><b>Links to tables</b></p>  <p>For example, use language of division linked to tables using counting stick</p> <p><b>Using known facts</b></p> <p>If <math>2 \times 3 = 6</math> then <math>200 \times 3 = 600</math> and <math>600 \div 3 = 200</math></p> <p><b>Rearranging the dividend to find multiples of the divisor.</b></p> <p><math>69 \div 3 =</math></p> <p>'What do I know about the 3 x tables?'</p> <p>"I know <math>3 \times 10 = 30</math> and <math>3 \times 3 = 9</math>."</p>  <p><math>69 \div 3 = 23</math></p>	<p><b>Place value materials to represent calculations</b></p> <p><b>Short division</b></p> <p><math>372 \div 6 =</math></p> $\begin{array}{r} 62 \\ 6 \overline{) 372} \\ \underline{36} \phantom{0} \\ 12 \\ \underline{12} \\ 0 \end{array}$ <p>'372 divided by 6. 3 hundreds cannot be shared equally between 6, so exchange the hundreds for 30 tens. I now have 37 tens which shared equally between 6 is 6 with a remainder of 1 ten. Exchange the ten for 10 units. I now have 12 ones which shared equally between 6 is 2. The answer is 62.'</p> <p><b>Representing problems and conditional knowledge</b></p>
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	 $48 \div 3 = 16$	<p>Andy says, 'I can use my three times table to work out <math>180 \div 3</math>'. Explain what Andy could do to work out this calculation.</p>	$2 \ 4 \ r \ 1$  <p>Remainders can never be greater than the divisor.</p>	<p>7 <input type="text"/> = <math>240 \div 8</math></p>  <p>8 <math>840 \div 5 =</math></p>  <p>Which calculations give 2 digit quotients?</p> <p> <math>36 \div 4</math>      <math>36 \div 3</math>      <math>40 \div 5</math>  <math>60 \div 6</math>      <math>80 \div 5</math> </p>
Known facts	Recall and use $\times$ and $\div$ facts for the 3, 4 and 8 $\times$ tables		Recall $\times$ and $\div$ facts for $\times$ tables up to $12 \times 12$ .	
Essential knowledge	Review division facts (2 $\times$ , 5 $\times$ and 10 $\times$ tables)	Halve 2 digit numbers	Division facts (4 $\times$ and 8 $\times$ tables)	10 $\times$ smaller
	Division facts (4 $\times$ table)	Division facts (3 $\times$ table)	Division facts (3 $\times$ , 6 $\times$ and 12 $\times$ tables)	Halve larger numbers and decimals
	Division facts (8 $\times$ table)	Division facts (6 $\times$ table)	Division facts (3 $\times$ and 9 $\times$ tables)	Division facts (11 $\times$ and 7 $\times$ tables)
Tests of divisibility	KSI: 2, 5, 10	Any number with a digit sum of a multiple of 3, will divide equally by 3	Any number with a digit sum of a multiple of 3, will divide equally by 3 KSI: 2, 5, 10	Any number with a digit sum of a multiple of 3 and is even will divide equally by 6

Year	5	6
<p>Layers of vocabulary</p>  <p>Appendix Ia Beck's Tiers of Vocabulary Appendix Ib: Vocabulary book</p>	<p><b>Basic to subject specific (Beck's Tiers):</b> equal groups of divide, division, divided by, divided into remainder factor, quotient, divisible by inverse</p> <p><b>Instructional vocabulary:</b> calculate, work out, solve, investigate question, answer, check</p> <p>same, different missing number/s number facts, number pairs, number bonds greatest value, least value</p> <p>NFER Arithmetic</p>	<p><b>Basic to subject specific (Beck's Tiers):</b> equal groups of divide, division, divided by, divided into remainder factor, quotient, divisible by inverse, remainders as fractions or decimals</p> <p><b>Instructional vocabulary:</b> calculate, work out, solve, investigate question, answer, check</p> <p>same, different missing number/s number facts, number pairs, number bonds greatest value, least value</p> <p>NFER Arithmetic</p>
<p>NC 2014</p>	<p>Divide numbers up to 4 digits by a 1 digit number using the formal written method of short division and interpret remainders appropriately for the context (as remainders, as fractions, as decimals or by rounding, e.g. <math>98 \div 4 = \frac{98}{4} = 24 \text{ r}2 = 24 \square = 24.5 \approx 25</math>).</p> <p>Solve problems involving multiplication and division including using knowledge of factors and multiples, squares and cubes. Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign. Solve problems involving multiplication and division including scaling by simple fractions and problems involving simple rates.</p>	<p>Divide numbers up to 4 digits by a 2 digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate to the context.</p> <p>Divide numbers up to 4 digits by a 2 digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.</p> <p>Solve problems involving addition, subtraction, multiplication and division.</p>

Developing declarative, procedural, conditional knowledge

**Using known facts**

If  $6 \div 2 = 3$  then  $6000 \div 2 = 3000$  and  
 $6000 \div 20 = 300$

Place value materials to represent calculations

**Short division**

$483 \div 7 =$

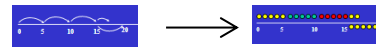
$$\begin{array}{r} 6 \\ 7 \overline{) 483} \\ \underline{42} \phantom{0} \\ 63 \\ \underline{63} \\ 0 \end{array}$$

"483 divided by 7. 4 hundreds cannot be shared equally between 7, so exchange the hundreds for 40 tens. I now have 48 tens which shared equally between 7 is 6 with a remainder of 6 tens. Exchange the 6 tens for

**Interpreting remainders**

$17 \div 5$

"What do I know? 17 is not a multiple of 5."

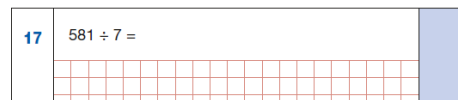


$3 \frac{2}{5} = 3.4$

5

From knowledge of decimal/fraction equivalents or by converting  $\frac{2}{5}$  into  $\frac{4}{10}$ .

**Conditional knowledge**



**Using known facts**

If  $6 \div 2 = 3$  then  $6 \div 0.2 = 30$  and

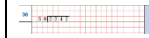
$6 \div 0.02 = 300$

**Short division**

$97.6 \div 5 =$

$$\begin{array}{r} 19.52 \\ 5 \overline{) 97.60} \\ \underline{5} \phantom{0} \\ 47 \\ \underline{45} \phantom{0} \\ 26 \\ \underline{25} \\ 10 \\ \underline{10} \\ 0 \end{array}$$

"97.6 divided by 5. 9 tens shared equally between 5 is 1 with a remainder of 4 tens. Exchange the ten for 10 ones. I now have 47 units which shared equally between 5 is 9 with a remainder of 2 ones. Exchange the 2 ones for 20 tenths, we now have 26 tenths. 26 shared equally between 5 equals 5 with a remainder of 1 tenth. Extend the dividend with a 0 in the hundredth's column. Exchange the tenth for 10



With questions of this type where the divisor is close to a number linked to the times tables, encourage the children to use known facts and adjustment to set up the partial tables.

		Adjust	
1	60	-1	59
2	120	-2	118
4	240	-4	236
5	300	-5	295
8	480	-8	472
10	600	-10	590

60 ones, we now have 64 ones. 64 shared equally between 7 equals 9 remainder 1. The answer is 69 r1."

$581 \div 7$  could be calculated by the formal written method of short division or it could be calculated by rearranging the dividend, using known facts, into 560 and 21.

**Representing problems**

Correct the errors in the calculation below. Explain the error.  $266 \div 5 = 73.1$

Alan says that the solution to  $186 \div 4$  can be written as '46 remainder 2' or as '46.5'. Do you agree? Explain your answer.

hundredths. 10 shared equally between 5 equals 2. The answer is 19.52."

**Long division – partial table using doubling and halving**



(thinking not generally recorded)

$384 \div 16$

1	16
2	32
4	64
5	80

"What do I know about the divisor?"

Record partial tables.

**Representing problems**

Megan divides 500 by 8 and gets the answer 62r4. She re writes it as  $62 \text{ r } 1/2$ . Is she right? Explain your answer.

**Conditonal Knowledge**

Using factors to simplify long division

$25 \overline{) 815}$

$\frac{165}{5 \overline{) 815}}$

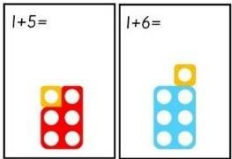
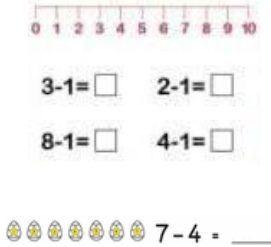
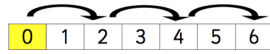

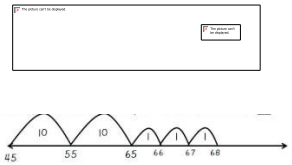
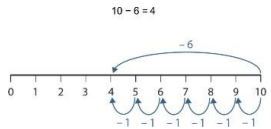
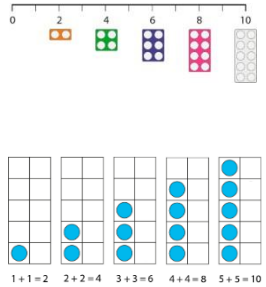

$\frac{35}{5 \overline{) 165}}$

Simplify the fractions for remainders

			<table border="1"> <tr> <td>8</td> <td>128</td> </tr> <tr> <td></td> <td></td> </tr> <tr> <td>10</td> <td>160</td> </tr> </table> $  \begin{array}{r}  24 \\  16 \overline{) 384} \\  \underline{32} \phantom{0} \\  64 \\  \underline{64} \\  0  \end{array}  $ <p>(38 tens <math>\div 16 = 2</math> r6; <math>2 \times 16 = 32</math>)      (bring the 4 down)      (64 units <math>\div 16 = 4</math>)      =      64      (no remainder)      0</p>	8	128			10	160	
8	128									
10	160									
Known facts	Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers. Recall prime numbers up to 19		Identify common factors, common multiples and prime numbers							
Essential knowledge	Division facts (4 x and 8 x tables)	100, 1000 times smaller	Division facts up to 12 x 12	Halve larger numbers and decimals						
	Division facts (3 x, 6 x and 12 x tables; 3 x and 9 x tables)	Partition to divide mentally	Apply place value to derive division facts, e.g. $12 \div 3 = 4$ so $1.2 \div 3 = 0.4$	Partition to divide mentally including decimals						
	Division facts (11 x and 7 x tables)	Halve larger numbers and decimals								

Tests of divisibility	Tests for 2,3,5,6 & 10	Any number with a digit sum of a multiple of 9 will divide equally by 9	Tests for 2,3,5,6, 9 & 10	Any number where the last two digits are divisible by 4, will all divide by 4
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## Standard written method ~ KS1 and KS2


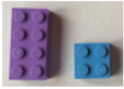

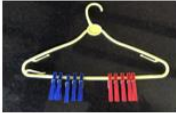






	Addition	Subtraction	Multiplication	Division
Reception				
Year 1				



Year 2	$\begin{array}{r} 59 \\ +43 \\ \hline 102 \end{array}$	$\begin{array}{r} 73 \\ -49 \\ \hline 24 \end{array}$	$8 \times 5 = 40$	$35 \div 5 = 7$
Year 3	$\begin{array}{r} 523 \\ +393 \\ \hline 916 \end{array}$	$\begin{array}{r} 523 \\ -393 \\ \hline 130 \end{array}$	$\begin{array}{r} 59 \\ \times 6 \\ \hline 354 \end{array}$	$\begin{array}{r} 4 \\ 8 \overline{)32} \end{array}$
Year 4	$\begin{array}{r} 1,312 \\ +3,094 \\ \hline 4,406 \end{array}$	$\begin{array}{r} 6,273 \\ -1,093 \\ \hline 5,180 \end{array}$	$\begin{array}{r} 159 \\ \times 16 \\ \hline 954 \\ +1,590 \\ \hline 2,544 \end{array}$	$\begin{array}{r} 135 \\ 7 \overline{)945} \end{array}$

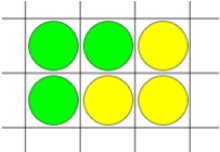
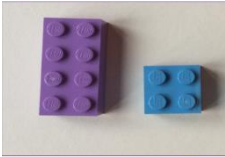
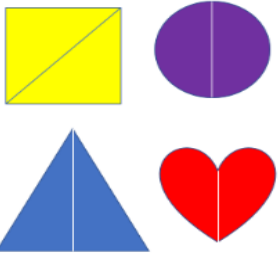
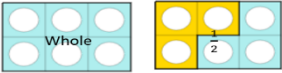
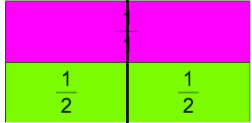
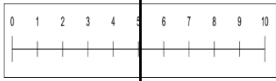
Year 5	<p>13,123</p> <p><u>30,943+</u></p> <p>44,066</p>	<p><sup>1</sup>62,<sup>1</sup>743</p> <p><u>10,923-</u></p> <p>51,820</p>	<p>2259</p> <p><u>6x</u></p> <p>54</p> <p>300</p> <p>1,200</p> <p><u>12,000+</u></p> <p>13,554</p>	<p>279 r 5</p> <p>6 <u>1679</u></p>
Year 6	<p>613,123</p> <p>1310,943+</p> <p>744,066</p>	<p>6112,1743</p> <p>100,923-</p> <p>511,820</p>	<p>2259</p> <p>4</p> <p>6x 13,554</p> <p>901,360+</p> <p>103,914</p>	<p>0389,739</p> <p>23 <u>8964</u></p> <p>23 <u>69</u></p> <p>46 <u>206</u></p> <p>69 <u>184</u></p> <p>92 <u>0224</u></p> <p>115 <u>207-</u></p> <p>138 <u>0170</u></p> <p>161 <u>161-</u></p> <p>184 <u>0090</u></p> <p>207 <u>69-</u></p> <p>210</p> <p><u>207-</u></p> <p>003</p>

Concrete	Pictorial	Continuous Provision
<p>Enjoys filling and emptying containers. Recognises that two objects have the same shape.</p>		
  <p data-bbox="450 906 651 963">Full, half full.</p> <p data-bbox="692 775 840 890">Half of the fruit.</p>	<p data-bbox="898 762 1373 799">Matching the other half pictures and jig saws.</p> 	<p data-bbox="1776 699 2074 762">Sharing fruit at snack time. Having half a glass of juice.</p>   <p data-bbox="2007 831 2168 930">Cut the toast in half, and half again.</p>
<p>Responds to and uses language of position and direction. Composes and decomposes shapes, learning which shapes combine to make other shapes.</p>		

 <p>The pegs are on half the leaf.</p>  <p>The blue brick is half the purple brick.</p>   <p>Find half the pegs.</p>	 <p>A square can be made from two triangles</p>  <p>Cut the leaf in half.</p> 	 <p>Would you like half an apple? Or half a satsuma?</p>  <p>Run half way round</p>  <p>Explore half the leaves.</p>
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## Progression in Fractions EYFS

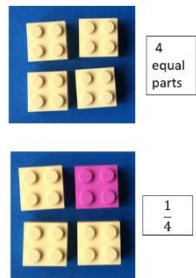
## Progression in Fractions KSI

Y1 Concrete	Y1 Pictorial	Y1 Abstract
Objective 1: Recognise, find and name a half as one of two equal parts of an object, shape or quantity.		
  <p>Whole      <math>\frac{1}{2}</math></p> <p>8      4</p>	 <p>How many ways can you show <math>\frac{1}{2}</math>?</p> 	 

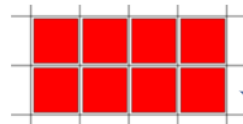
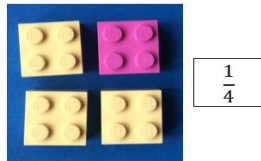
$$\frac{1}{2} \text{ of } 8 = 4$$



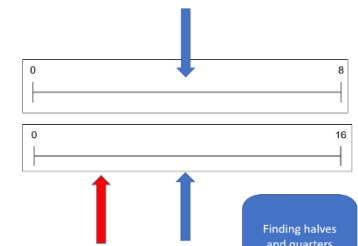
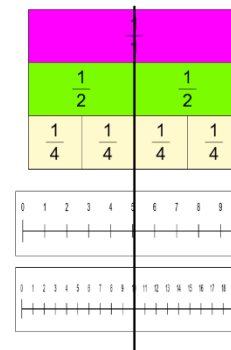
Objective 2: Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.



How many ways can you show  $\frac{1}{4}$ ?



How many different ways can you show  $\frac{1}{4}$  of these counters?



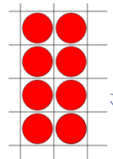
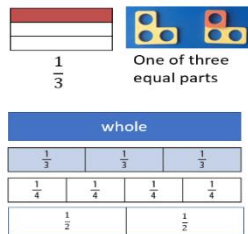
Finding halves and quarters

Y2 Concrete

Y2 Pictorial

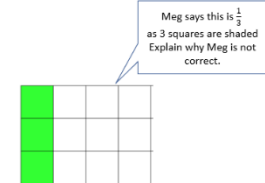
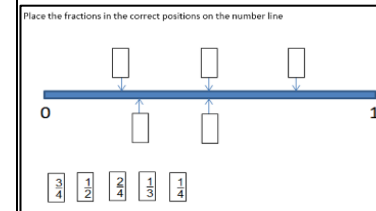
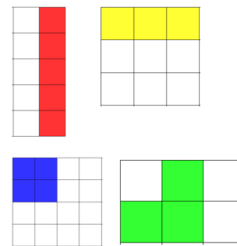
Y2 Abstract

Objective 1: Recognise, find, name and write fractions  $\frac{1}{3}$ ,  $\frac{1}{2}$ ,  $\frac{2}{4}$  and  $\frac{1}{4}$  of a length, shape, set of objects or quantities.



How many different ways can you show  $\frac{1}{2}$  and  $\frac{1}{4}$  of these counters?

Write the fraction that is shaded.

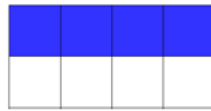


Meg says this is  $\frac{1}{3}$  as 3 squares are shaded. Explain why Meg is not correct.

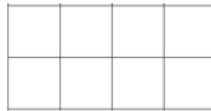
Objective 2: Write simple fractions and recognise the equivalence of  $\frac{1}{2}$  and  $\frac{2}{4}$



$$\frac{1}{2} = \frac{2}{4}$$

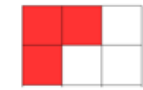


$$\frac{1}{2} \text{ of } 8 = 4$$



Show that

$$\frac{1}{2} = \frac{2}{4}$$

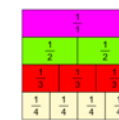
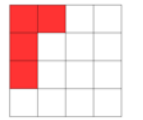


$$\frac{1}{2} \text{ of } 6 = 3$$

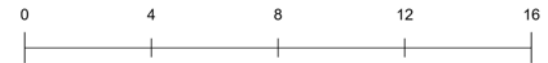


$$\frac{1}{2} \text{ of } 8 = 4$$

$$\frac{2}{4} \text{ of } 8 = 4$$



Odd one out



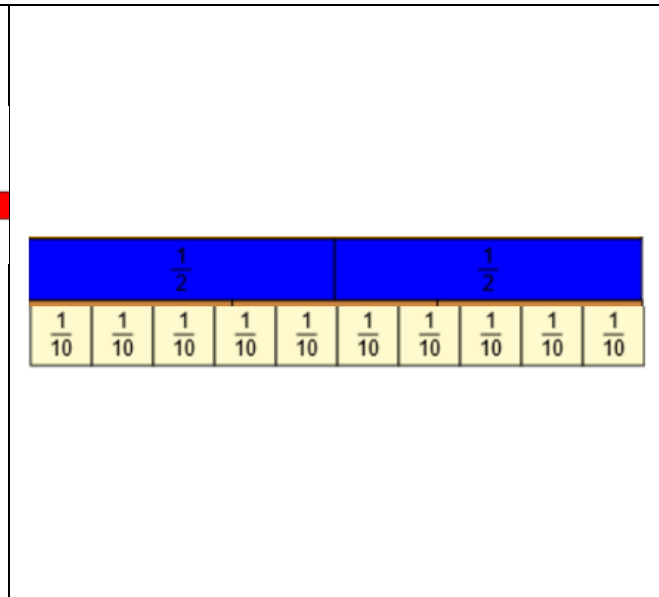
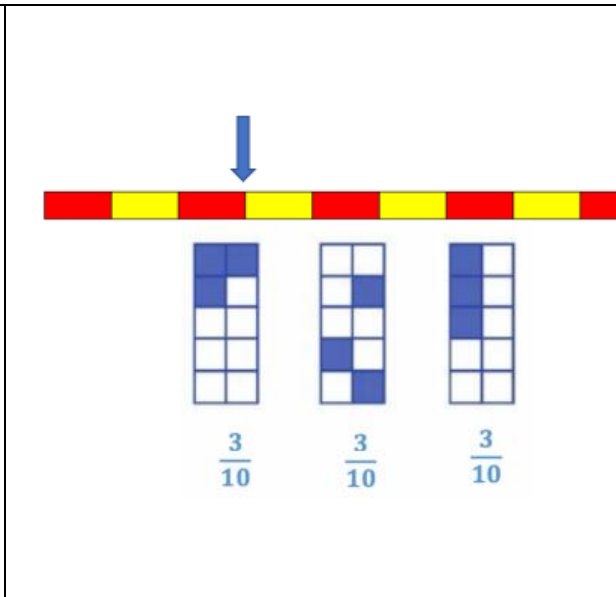
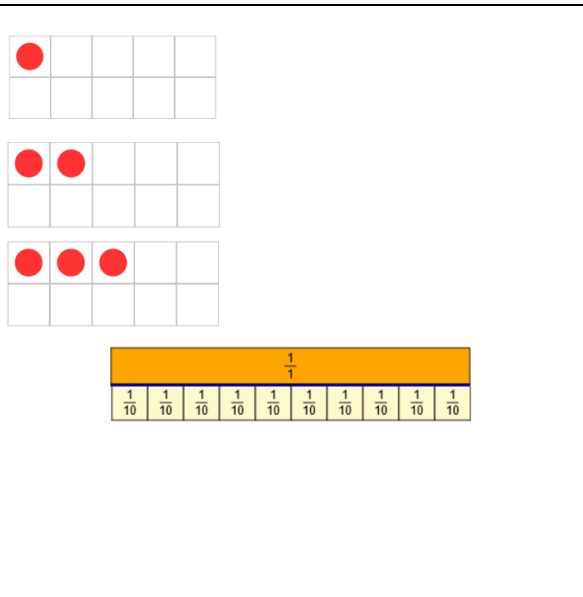
### Progression in Fractions LKS2

Y3 Concrete

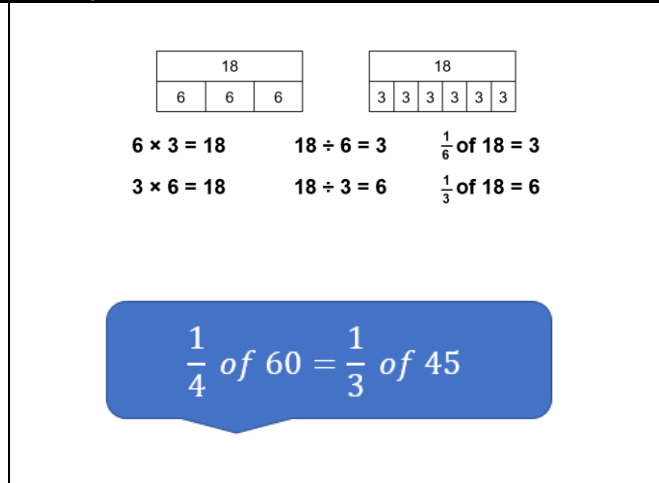
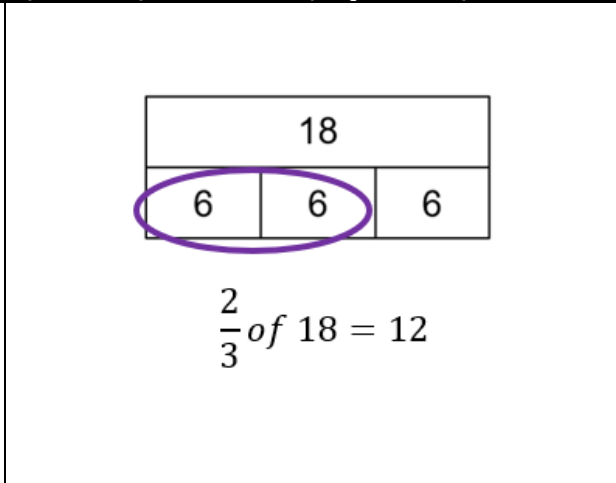
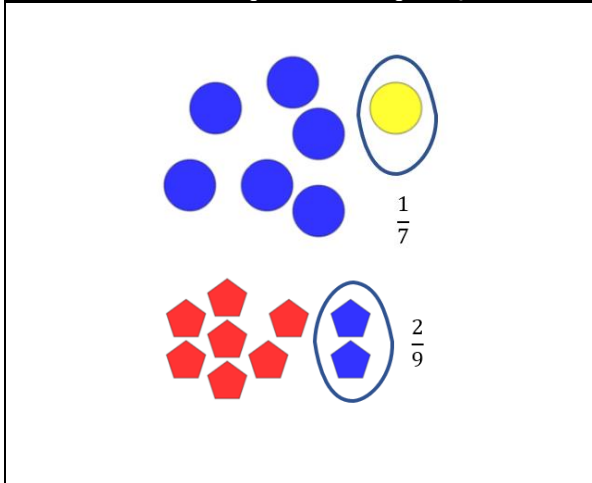
Y3 Pictorial

Y3 Abstract

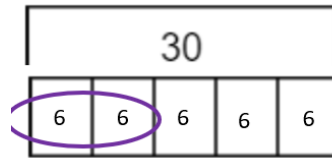
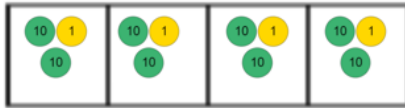
Objective 1: Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10.



Objective 2: Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.



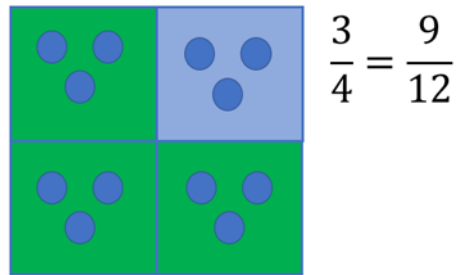
Find  $\frac{1}{4}$  of 84



$$\frac{2}{5} \text{ of } 30 = 12$$

$$\frac{2}{5} \text{ of } 60 = \frac{1}{2} \text{ of } 48$$

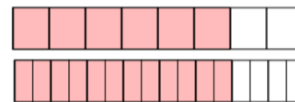
Objective 3: Recognise and show, using diagrams, equivalent fractions with small denominators.



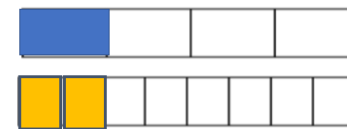
$$\frac{3}{4} = \frac{9}{12}$$



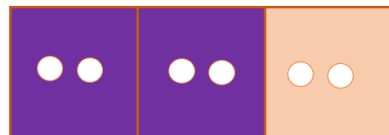
Show quarters and eighths



$$\frac{6}{8} = \frac{12}{16} = \frac{3}{4}$$



$$\frac{1}{4} \stackrel{\times 2}{=} \frac{2}{8}$$



$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{4}{5} \stackrel{\times 2}{=} \frac{8}{10}$$

$$\frac{3}{6} \stackrel{\times 5}{=} \frac{15}{30}$$

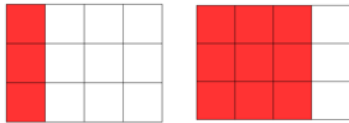
$$\frac{3}{9} \stackrel{\times ?}{=} \frac{6}{18}$$

Objective 4: Add and Subtract fractions with the same denominator within one whole.

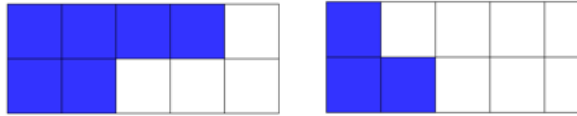




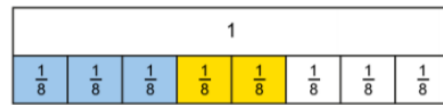
$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$



$$\frac{1}{4} + \frac{3}{4} = \frac{4}{4}$$



$$\frac{6}{10} - \frac{3}{10} = \frac{3}{10}$$



$$\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$$

$$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$$



Write three fraction equations for this model.

Answer

$$\frac{3}{8} + \frac{4}{8} =$$

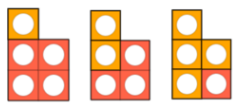
$$\frac{5}{7} - \frac{2}{7} =$$

True or false?

$$\frac{5}{6} + \frac{2}{6} = \frac{7}{12}$$

$$\frac{13}{20} - \frac{3}{20} = \frac{1}{2}$$

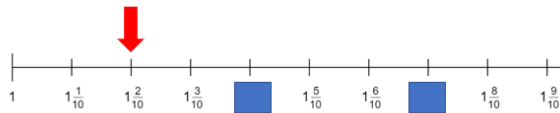
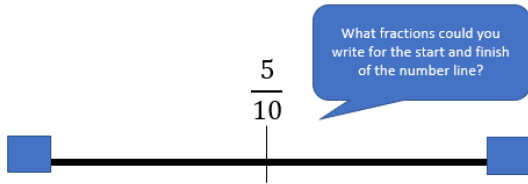
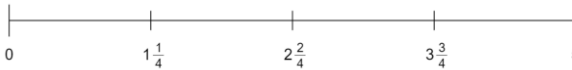
Objective 5: Compare and order unit fractions, and fractions with the same denominators.



$$\frac{1}{5}$$

$$\frac{2}{5}$$

$$\frac{4}{5}$$



Put  $<$ ,  $>$  or  $=$  in the circle to make this statements true.

$$\frac{6}{7} \bigcirc \frac{2}{7}$$

$$\frac{5}{10} \bigcirc \frac{8}{16}$$

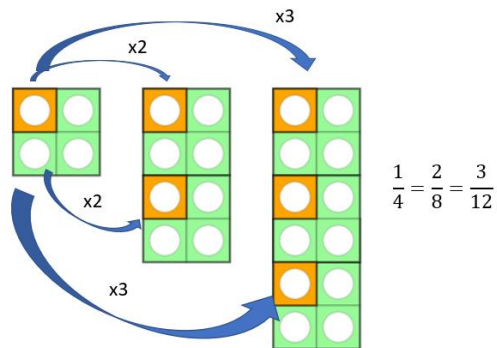
$$\frac{6}{15} \bigcirc \frac{3}{15}$$

Y4 Concrete

Y4 Pictorial

Y4 Abstract

Objective 1: Recognise and show, using diagrams, families of common equivalent fractions.



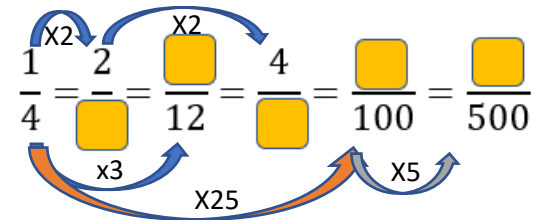
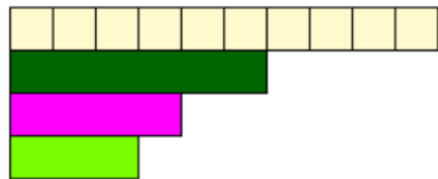
$$\frac{1}{2} \stackrel{\times 3}{=} \frac{3}{6}$$

$$\frac{1}{2} = \frac{6}{12}$$

$$\frac{1}{3} \stackrel{\times 6}{=} \frac{4}{12}$$

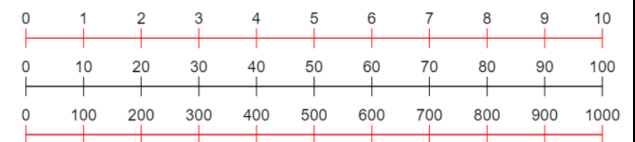
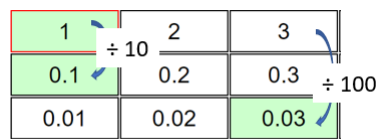
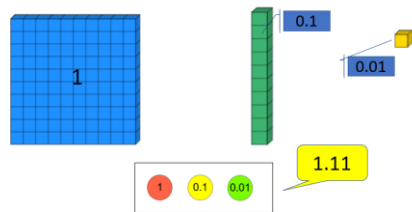
$$\frac{1}{4} \stackrel{\times 4}{=} \frac{3}{12}$$

$$\frac{1}{3}$$

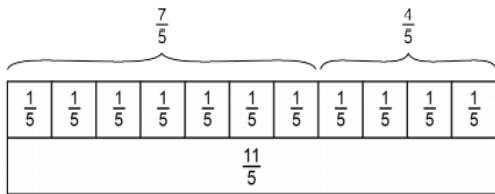


×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36

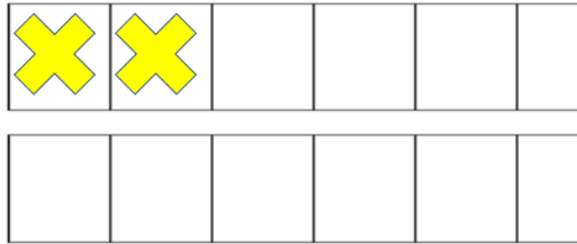
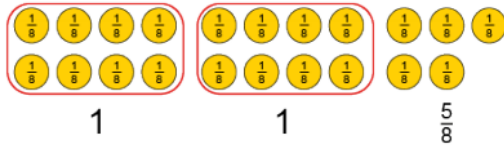
Objective 2: Count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.



Objective 3: Add and subtract fractions with the same denominator.



$$\frac{7}{5} + \frac{4}{5} = \frac{11}{5}$$



$$2 - \frac{2}{6} = \frac{10}{6}$$

$$\frac{7}{8} + \frac{3}{8} =$$

$$4 + \frac{6}{8} =$$

$$3 - \frac{5}{6} =$$

$$\frac{7}{5} + \frac{4}{5} = \frac{11}{5}$$

$$3\frac{7}{8} - \frac{2}{8} = 3\frac{5}{8}$$

Draw models to prove these statements are true

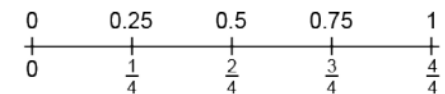
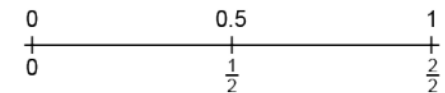
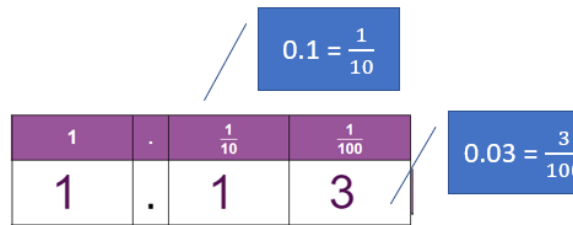
$$7\frac{2}{5} + \frac{4}{5} = 8\frac{1}{5}$$

$$8\frac{1}{5} - \frac{4}{5} = 7\frac{2}{5}$$

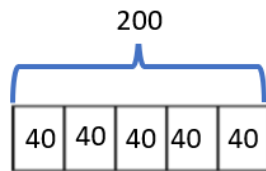
Objective 4: Recognise and write decimal equivalents of any number of tenths or hundredths.

Unit fraction	Decimal fraction
$\frac{1}{2}$	0.5
$\frac{1}{4}$	0.25
$\frac{1}{5}$	0.2
$\frac{1}{10}$	0.1

$$\frac{1}{4} = \frac{25}{100} = 0.25$$

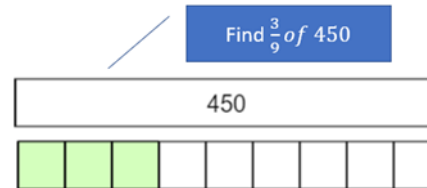
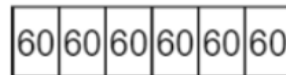
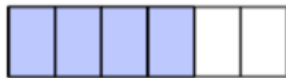


Objective 5: Solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number

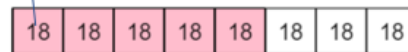


$$\frac{3}{5} \text{ of } 200 = 120$$

$$\frac{4}{6} \text{ of } 360 = 240$$



Write an equation for this model



Judy and Tilly each have a 1m ribbon. Judy cuts her ribbon into 10 equal parts and uses 1 of them to make a hair tie. Tilly cuts her ribbon into 10 equal parts and uses 3 of them to make a hair tie. Have Judy and Tilly used the same amount of ribbon? Explain your answer.

Petrol cans hold 2 litres.

Tony uses  $\frac{3}{4}$  of a 2 litre can of petrol in his lawn mower. He then uses  $\frac{1}{2}$  of another can with  $\frac{6}{8}$  remaining of the 2 litres.

How much petrol did he use altogether?

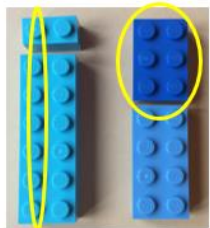
## Progression in Fractions UKS2

Y5 Concrete

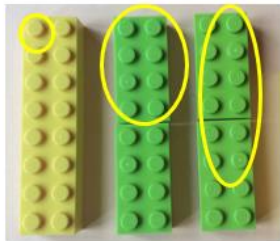
Y5 Pictorial

Y5 Abstract

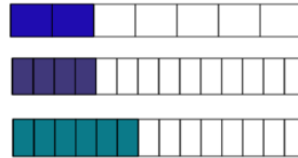
Objective 1: Compare and order fractions whose denominators are all multiples of the same number.



$$\frac{1}{7} \quad \frac{6}{14} = \frac{3}{7}$$



$$\frac{1}{16} \quad \frac{1}{2} \quad \frac{3}{4} = \frac{6}{8}$$



$$\frac{2}{7} = \frac{4}{14} \quad \frac{6}{14} = \frac{3}{7}$$



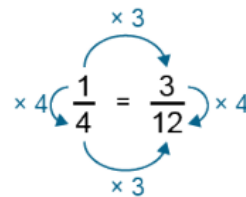
$$\frac{3}{8}$$



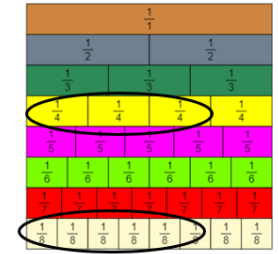
$$\frac{7}{16}$$



$$\frac{12}{16} = \frac{3}{4}$$



x	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16
5	5	10	15	20
6	6	12	18	24
7	7	14	21	28

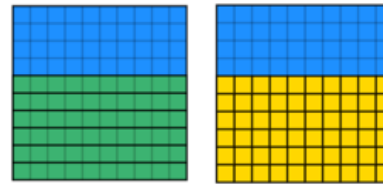
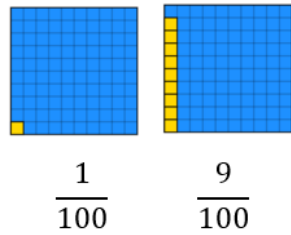
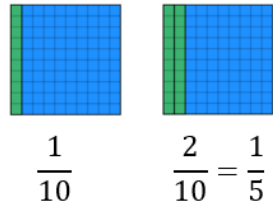


$$\frac{3}{4} = \frac{6}{8}$$

Complete the table to show three more fractions that are equivalent to the first:

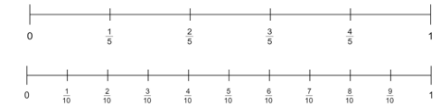
Fraction	Equivalent fractions		
$\frac{1}{2}$	$\frac{2}{4}$		
$\frac{1}{4}$	$\frac{2}{8}$		
$\frac{1}{3}$			

Objective 2: Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths.



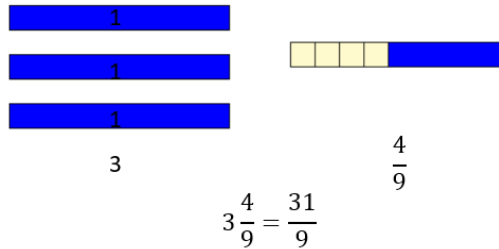
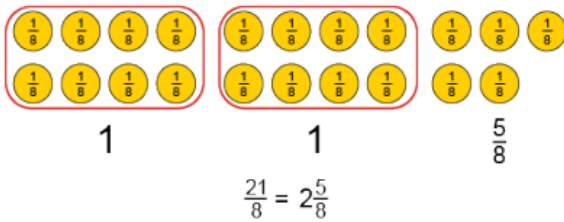
$$\frac{6}{10} = \frac{60}{100}$$

$$\frac{6}{10} = \frac{60}{100}$$



$$\frac{4}{10} = \frac{2}{5}$$

Objective 3: Recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements  $> 1$  as a mixed number.



Express the following improper fractions as mixed numbers.

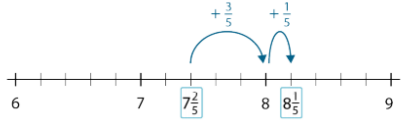
$$\frac{17}{2} \quad \frac{13}{6} \quad \frac{28}{10} \quad \frac{41}{7}$$

Express the following mixed numbers as improper fractions.

$$4\frac{1}{8} \quad 6\frac{4}{9} \quad 3\frac{11}{12} \quad 8\frac{2}{3}$$

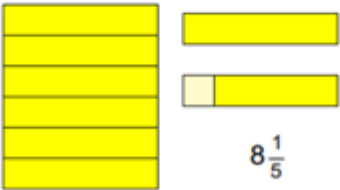
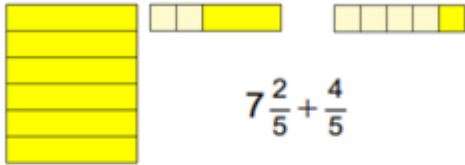
Objective 4: Add and subtract fractions with the same denominator and denominators that are multiples of the same number.

adding to reach the whole number, then adding the remaining fraction



$$7\frac{2}{5} + \frac{4}{5} = 8\frac{1}{5}$$

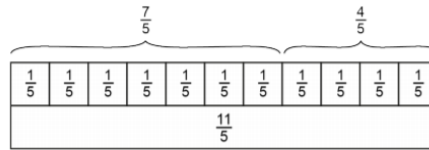
$$8\frac{1}{5} - \frac{4}{5} = 7\frac{2}{5}$$



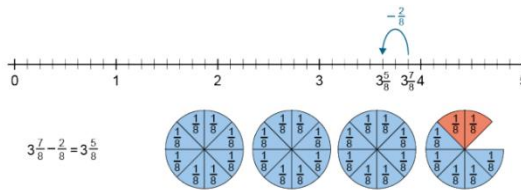
$$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1\frac{2}{5}$$

**Language focus**

"7 one-fifths plus 4 one-fifths is equal to 11 one-fifths."



$$\frac{7}{5} + \frac{4}{5} = \frac{11}{5}$$



$$3\frac{7}{8} - \frac{2}{8} = 3\frac{5}{8}$$

1. It is a  $2\frac{3}{4}$  km cycle ride to my friend's house, and a further  $\frac{3}{4}$  km ride to the park. How far do I have to cycle altogether?

2. I have 5m of rope. I cut off  $\frac{4}{10}$  m. How much rope is left?

3. Fill in the missing numbers.



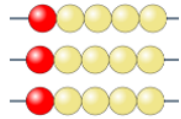
4. The table below shows the number of hours Josie read each day during a school week. For how long did Josie read altogether?

Mon	Tues	Wed	Thurs	Fri
$1\frac{3}{4}$ hours	1 hour	$1\frac{1}{4}$ hours	$1\frac{1}{4}$ hours	$2\frac{3}{4}$ hours

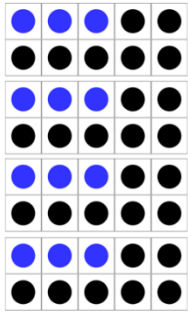
5. A tailor has  $3\frac{7}{10}$  m of ribbon. She uses  $1\frac{9}{10}$  m to complete a dress. How much ribbon is left?

Objective 5: Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.

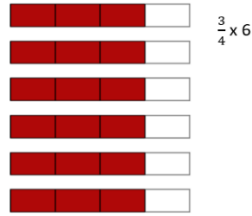




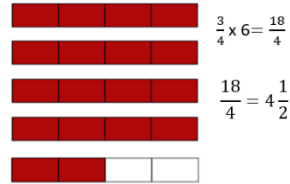
$$\frac{1}{5} \times 3 = \frac{3}{5}$$



$$\frac{3}{10} \times 4 = \frac{12}{10} = 1\frac{2}{5}$$



$$\frac{3}{4} \times 6$$



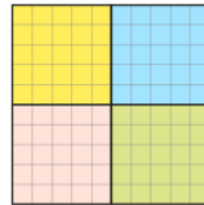
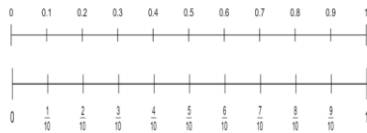
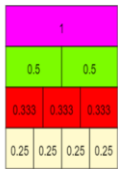
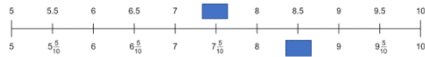
$$\frac{3}{4} \times 6 = \frac{18}{4}$$

$$\frac{18}{4} = 4\frac{1}{2}$$

$$\frac{1}{2} \times 5 = \frac{1}{2} \times \frac{5}{1}$$

$$\frac{3}{5} \times 4 = \frac{3}{5} \times \frac{4}{1} = \frac{12}{5} = 2\frac{2}{5}$$

Objective 6: Read and write decimal numbers as fractions [for example, 0.71 = 71/100]



$$\frac{1}{4} = \frac{25}{100} = 0.25$$

Fill in the missing symbols (<, > or =).

$$\frac{1}{10} \square 0.75$$

$$0.4 \square \frac{1}{4}$$

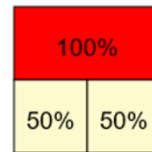
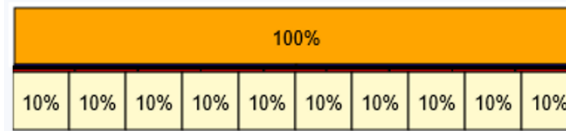
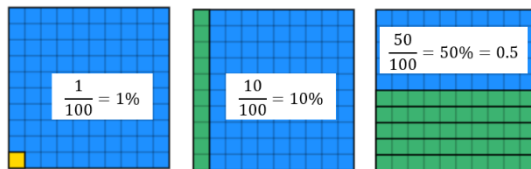
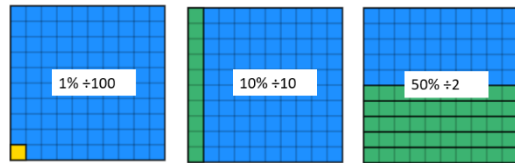
$$0.5 \square \frac{1}{5}$$

$$\frac{3}{4} \square 0.75$$

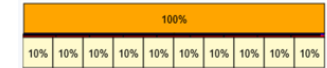
$$0.8 \square \frac{4}{5}$$

$$\frac{1}{2} \square 0.2$$

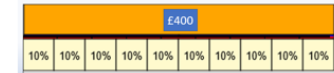
Objective 7: Recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal.



100% = 500  
 What is 10%?  
 What is 50%  
 Is 70% = 350



100% = £400  
 What is 30%?  
 Is 20% = £90?



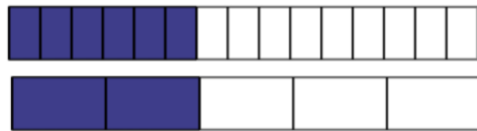
100%	50%	25%	75%	10%
1000 m	500 m	250 m	750 m	100 m
160 kg	80 kg	40 kg	120 kg	16 kg
1080 mm	540 mm	270 mm	810 mm	108 mm

Y6 Concrete

Y6 Pictorial

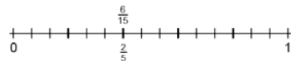
Y6 Abstract

Objective 1: Use common factors to simplify fractions; use common multiples to express fractions in the same denominator.



$$\frac{6}{15} = \frac{2}{5}$$

+3  
-3



$$\frac{4}{12} = \frac{1}{3}$$

+4  
-4



×	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

$$\frac{4}{12} = \frac{1}{3}$$

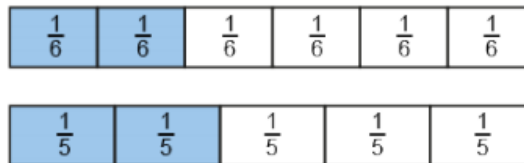
÷4  
÷4

Sort these fractions according to whether they are expressed in their simplest form or not.

$\frac{3}{15}$   $\frac{2}{5}$   $\frac{4}{20}$   $\frac{25}{36}$   $\frac{1}{6}$   $\frac{7}{21}$   $\frac{18}{30}$   $\frac{9}{17}$   $\frac{5}{15}$   $\frac{11}{20}$   $\frac{23}{30}$

Fraction in its simplest form	Fraction <u>not</u> in its simplest form

Objective 2: Compare and order fractions, including fractions > 1.



$$\frac{2}{5} > \frac{2}{6}$$

$$\frac{2}{5} = \frac{12}{30}$$

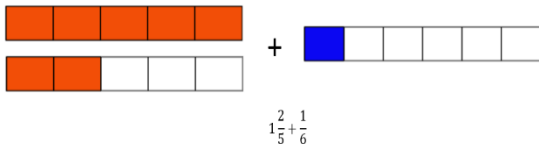
$$\frac{2}{6} = \frac{10}{30}$$

LCM of 5 and 6 = 30

Without using a common denominator, put each set of fractions in order from smallest to largest.

a.  $\frac{10}{8}$   $\frac{7}{8}$   $\frac{5}{8}$   $\frac{3}{8}$   $\frac{8}{8}$   $\frac{4}{8}$   $\frac{2}{8}$

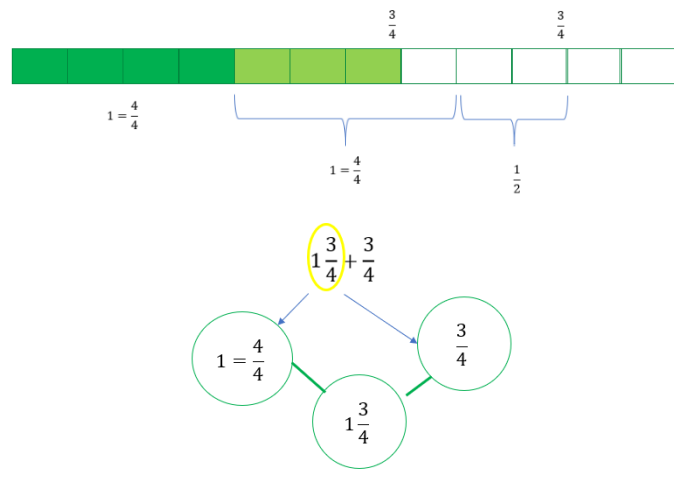
Objective 3: Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.



$$\frac{7}{5} + \frac{1}{6}$$

$$\frac{42}{30} + \frac{5}{30} = \frac{47}{30}$$

$$\frac{47}{30} = 1 \frac{17}{30}$$



$$2 \frac{1}{2} + \frac{3}{4} =$$

$$\frac{7}{10} + \frac{5}{10} + \frac{3}{10}$$

$$\frac{13}{8} + \frac{11}{8}$$

$$\frac{6}{7} + \frac{2}{3} = \frac{18}{21} + \frac{14}{21}$$

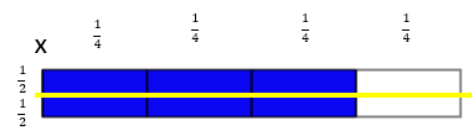
$$\frac{8}{9} + \frac{8}{9} - \frac{1}{9}$$

$$7 \frac{1}{6} - 1 \frac{2}{6}$$

$$3 \frac{7}{10} + 2 \frac{9}{10}$$

$$\frac{17}{3} - \frac{5}{3}$$

Objective 4: Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example,  $1/2 \times 3/4 = 3/8$ ].



$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

$$\frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$$

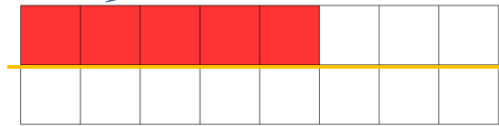
Objective 5: Divide proper fractions by whole numbers [for example,  $6/8 \div 3 = 1/4$ ]

Dividing by 2 is the same as multiplying by  $\frac{1}{2}$



$$\frac{5}{8} \div 2 =$$

The denominator is now 16.

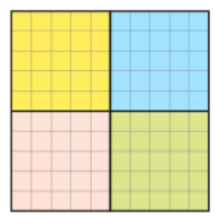


The reciprocal:  
 $\frac{1}{8} \div 2 = \frac{1}{8} \div \frac{2}{1}$   
 $\frac{1}{8} \times \frac{1}{2} = \frac{1}{16}$

Objective 6: Associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, 3/8].



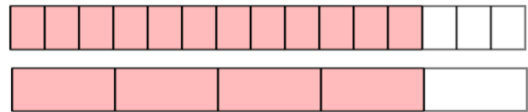
1.25



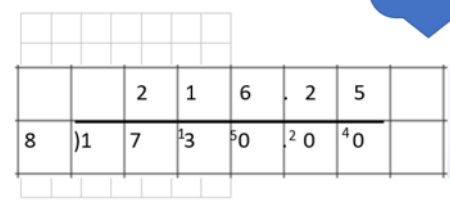
$$\frac{1}{4} = \frac{25}{100} = 0.25$$

$$\begin{array}{r} 28 \\ 15 \overline{) 432} \\ \underline{30} \phantom{0} \\ 132 \\ \underline{120} \\ 12 \end{array} \quad \begin{array}{l} (15 \times 2) \\ (15 \times 8) \end{array} \quad 28 \frac{12}{15} = \frac{4}{5}$$

$$28 \frac{4}{5} = 28.8$$



$$1730 \div 8$$



216 r 2  
 $\frac{2}{8}$   
 $\frac{1}{4}$

